# ST406-15 Applied Stochastic Processes with Advanced Topics

## 26/27

**Department**
Statistics
**Level**
Undergraduate Level 4
**Module leader**
Daniel Valesin
**Credit value**
15
**Module duration**
10 weeks
**Assessment**
Multiple
**Study location**
University of Warwick main campus, Coventry

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## Description

## Introductory description

This module is available for students on a course where it is a listed option and as an Unusual Option to students who
have the required backgrounds as covered by the pre-requisite module.

The ideas presented in this module have a vast range of applications, for example routing algorithms in telecommunications (queues), assessment of apparent spatial order in astronomical data (stochastic geometry), description of outbreaks of disease (epidemics). We will only be able to introduce each area - indeed each area could easily be the subject of a course on its own! But the introduction will provide you with a good base to follow up where and when required. (For example: a MORSE graduate found that their firm was asking them to address problems in queuing theory, for which ST333 provided the basis.) We will discuss these and other applications and show how the ideas of stochastic process theory help in formulating and solving relevant questions.

Students will be given selected advanced research material for independent study and examination.

**Pre-requisites:** ST227 Stochastic Processes.

**Anti-requisites:** For anti-requisites please check under the availability tab and in the course handbook.

Module web page

## Module aims

To provide an introduction to concepts and techniques which are fundamental in modern applied probability theory and operations research:
Models for queues, point processes, and epidemics.
Notions of equilibrium, threshold behaviour, and description of structure.

Students will be given selected advanced research material for independent study and examination.

## Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Continuous time Markov Chains.
   Terms used in the analysis of continuous-time Markov chains: Markov property, transition probability function, standing assumptions, Chapman-Kolmogorov equations, Q-matrix, Kolmogorov forward and backward differential equations, equilibrium distribution. The simplest case: finite state-space Markov chains. The "switcher" example. Exact transition densities for processes on a small number of states. The strong Markov property.

2. Linear Birth-Death processes.
   Poisson (counting) process: construction, ideas of independent increments, superposition, counts and thinning. Pure birth process, linear birth-death process, birth-death-immigration process: construction using "microscopic model", derivation of extinction and equilibrium probabilities. Generalized birth-death processes.

3. Queuing theory.
   The Markov single-server (M/M/1) queue. The concept of detailed balance. Measures of effectiveness. Multiserver (M/M/cl/c2) queues. Erlang's formula. Queues with general service-time distribution (M/G/l) and their embedded Markov chains. Little's formula, Pollaczek-Khintchine formula.

4. Other Markov properties.
   Stopping times. Strong Markov property. Holding theorem.

5. Epidemics.
   Deterministic Epidemic model. Stochastic model without removals. Stochastic model with removals.

The module's advanced topic will include material for independent study and assessment. This material will reflect current trends, innovations or expertise in the area.

## Learning outcomes

By the end of the module, students should be able to:

- Be able to formulate continuous-time Markov chain models for applied problems.
- Be able to use basic theory to gain quick answers to important questions (for example, what is the equilibrium distribution for a specific reversible Markov chain?).
- Be able to solve for the transition probabilities for Markov chains on a finite state space.
- Understand how to use Markov chains in the modelling and analysis of queues and epidemics.
- Understand, through independent study, material on an advanced topic.

## Indicative reading list

Specific reading list for the module

## Subject specific skills

- Demonstrate facility with rigorousprobabilistic methods.
- Evaluate, select and apply appropriate mathematical and/or probabilist techniques.
- Demonstrate knowledge of and facility with formal probability concepts, both explicitly and by applying them to the solution of mathematical problems.
- Create structured and coherent arguments communicating them in written form.
- Construct logical arguments with clear identification of assumptions and conclusions.
- Reason critically, carefully, and logically.

## Transferable skills

- Problem solving: Use rational and logical reasoning to deduce appropriate and well-reasoned conclusions. Retain an open mind, optimistic of finding solutions, thinking laterally and creatively to look beyond the obvious. Know how to learn from failure.
- Self awareness: Reflect on learning, seeking feedback on and evaluating personal practices, strengths and opportunities for personal growth.
- Communication: Present arguments, knowledge and ideas, in a range of formats.
- Professionalism: Prepared to operate autonomously. Aware of how to be efficient and resilient. Manage priorities and time. Self-motivated, setting and achieving goals, prioritising tasks.

## Study

## Study time

| Type | Required |
|---|---|
| Lectures | 30 sessions of 1 hour (20%) |
| Seminars | 5 sessions of 1 hour (3%) |
| Private study | 115 hours (77%) |
| Total | 150 hours |

## Private study description

Study of advanced topic, weekly revision of lecture notes and materials, wider reading, practice exercises and preparing for examination.

### Other activity description

Revision support equivalent to approximately 2 hours.

## Costs

No further costs have been identified for this module.

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## Assessment

You must pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

### Assessment group B7

| | Weighting | Study time | Eligible for self-certification |
|---|---|---|---|
| Centrally-timetabled examination (On-campus) | 100% | | No |

The examination will contain one compulsory question on the advanced topic and four additional questions of which the best marks of TWO questions will be used to calculate your grade.

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- Answerbook Pink (12 page)
- Students may use a calculator

### Assessment group R5

| | Weighting | Study time | Eligible for self-certification |
|---|---|---|---|
| In-person Examination - Resit | 100% | | No |

The examination will contain one compulsory question on the advanced topic and four additional questions of which the best marks of TWO questions will be used to calculate your grade.

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- Answerbook Pink (12 page)
- Students may use a calculator

### Feedback on assessment

Solutions and cohort level feedback will be provided for the examination.

Past exam papers for ST406

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## Availability

### Anti-requisite modules

If you take this module, you cannot also take:

- ST333-15 Applied Stochastic Processes

## Courses

This module is Optional for:

- TMAA-G1PE Master of Advanced Study in Mathematical Sciences
  - Year 1 of G1PE Master of Advanced Study in Mathematical Sciences
  - Year 1 of G1PE Master of Advanced Study in Mathematical Sciences
- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
  - Year 1 of G1PC Mathematics (Diploma plus MSc)
  - Year 2 of G1PC Mathematics (Diploma plus MSc)
- Year 1 of TSTA-G4P1 Postgraduate Taught Statistics
- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics

    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics

This module is Option list A for:

- TSTA-G4P1 Postgraduate Taught Statistics
    - Year 1 of G4P1 Statistics (Taught)
    - Year 1 of G40A Statistics with Probability (Taught)
- Year 5 of USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
- USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
    - Year 3 of G1G3 Mathematics and Statistics (BSc MMathStat)
    - Year 4 of G1G3 Mathematics and Statistics (BSc MMathStat)
- USTA-G140 Undergraduate Mathematics and Statistics (MMathStat)
    - Year 3 of G1G3 Mathematics and Statistics (BSc MMathStat)
    - Year 4 of G1G3 Mathematics and Statistics (BSc MMathStat)

This module is Option list B for:

- TSTA-G4P1 Postgraduate Taught Statistics
    - Year 1 of G40B Statistics with Data Science (Taught)
    - Year 1 of G40C Statistics with Finance (Taught)
- Year 4 of USTA-G304 Undergraduate Data Science (MSci)
- Year 4 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 5 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 3 of USTA-G140 Undergraduate Mathematics and Statistics (MMathStat)

This module is Option list C for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
    - Year 4 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad

This module is Option list D for:

- Year 5 of USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated

This module is Option list E for:

- Year 5 of USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated

This module is Option list F for:

- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
- Year 4 of USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated