ST403-15 Brownian Motion

26/27

Department
Statistics
Level
Undergraduate Level 4
Module leader
Elke Thonnes
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

In 1827 the Botanist Robert Brown reported that pollen suspended in water exhibit random erratic movement. This ‘physical’ Brownian motion can be understood via the kinetic theory of heat as a result of collisions with molecules due to thermal motion. The phenomenon has later been related in Physics to the diffusion equation, which led Albert Einstein in 1905 to postulate certain properties for the motion of an idealized ‘Brownian particle’ with vanishing mass:

- the path t -»B(t) of the particle should be continuous;
- the displacements B(s+t)-B(s) should be independent of the past motion, and have a Gaussian distribution with mean 0 and variance proportional to t

Prerequisites:

- **UG Students:** ST318 Probability Theory OR ST350 Measure Theory for Probability OR MA359 Measure Theory.
- **MSc in Statistics Students:** ST964 Introduction to Advanced Probability
- **MSc in Mathematical Finance Students:** ST908 Stochastic Calculus for Finance.

Module web page

Module aims

The module studies the construction and properties of Brownian motion, a fundamental tool for modelling processes which evolve randomly in time. Brownian motion is used widely in many areas of pure and applied mathematics and in the last few decades it has become essential to the study of financial maths as a model of stock prices.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Topics discussed in this module include:

- construction of Brownian motion/Wiener process
- fractal properties of the path, which is continuous but still a rough, non-smooth function
- description as a Gaussian process, an important class of models in machine learning
- description as a Markov process in terms of generators and semigroups
- the martingale property of Brownian motion and some aspects of stochastic calculus
- scaling properties and connection to random walk
- connection to the Dirichlet problem, harmonic functions and PDEs
- some generalizations, including e.g. geometric Brownian motion and fractional Brownian motion

Learning outcomes

By the end of the module, students should be able to:

- Apply the martingale property of Brownian Motion (BM) to derive advanced properties such as Wald's lemmas.
- Construct and explain properties of BM.
- Apply BM as a continuous time and continuous state Markov process.
- Apply the embedding of random walks in Brownian motion and use it to derive convergence results.
- Translate properties of one-dimensional BM to higher dimensions.

Indicative reading list

Reading lists can be found in Talis

Specific reading list for the module

Subject specific skills

- describe its construction and explain simple properties of Brownian Motion (BM);
- understand BM as a continuous time and continuous state Markov process;

- use the martingale property of BM to derive advanced properties such as Wald’s lemmas;
- understand the embedding of random walks in Brownian motion and use it to derive convergence results;
- translate properties of one-dimensional BM to higher dimensions.

Transferable skills

- Problem solving: Use rational and logical reasoning to deduce appropriate and well-reasoned conclusions. Retain an open mind, optimistic of finding solutions, thinking laterally and creatively to look beyond the obvious. Know how to learn from failure.
- Self awareness: Reflect on learning, seeking feedback on and evaluating personal practices, strengths and opportunities for personal growth.
- Communication: Present arguments, knowledge and ideas, in a range of formats.
- Professionalism: Prepared to operate autonomously. Aware of how to be efficient and resilient. Manage priorities and time. Self-motivated, setting and achieving goals, prioritising tasks.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Review lectured material and work on set exercises.

Other activity description

Revision support.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group D6

	Weighting	Study time	Eligible for self-certification
Assignments worth 15% Coursework	15%		No
Centrally-timetabled examination (On-campus)	85%		No

The examination paper will contain four questions, of which the best marks of THREE questions will be used to calculate your grade.

- Answerbook Pink (12 page)

Assessment group R5

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

The examination paper will contain four questions, of which the best marks of THREE questions will be used to calculate your grade.

- Answerbook Pink (12 page)

Feedback on assessment

Assignments are marked and given feedback online within 20 working days of the submission deadline. Where
appropriate, model solutions will be provided.

Solutions and cohort level feedback will be provided for the examination.

Past exam papers for ST403

Availability

Anti-requisite modules

If you take this module, you cannot also take:

- MA4F7-15 Brownian Motion

Courses

This module is Optional for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- TIBS-N3G1 Postgraduate Taught Financial Mathematics
    - Year 1 of N3G1 Financial Mathematics
    - Year 1 of N3G1 Financial Mathematics
- Year 1 of TMAA-G1P9 Postgraduate Taught Interdisciplinary Mathematics
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 3 of G30A Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream)
    - Year 3 of G30J Master of Maths, Op.Res, Stats & Economics (Data Analysis Stream)
    - Year 3 of G30B Master of Maths, Op.Res, Stats & Economics (Econometrics and Mathematical Economics Stream)
    - Year 3 of G30C Master of Maths, Op.Res, Stats & Economics (Operational Research and Statistics Stream)
    - Year 3 of G30C Master of Maths, Op.Res, Stats & Economics (Operational Research and Statistics Stream)
    - Year 3 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Core option list A for:

- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 4 of G30A Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream)
    - Year 4 of G30J Master of Maths, Op.Res, Stats & Economics (Data Analysis Stream)
    - Year 4 of G30B Master of Maths, Op.Res, Stats & Economics (Econometrics and

Mathematical Economics Stream)
    - Year 4 of G30C Master of Maths, Op.Res, Stats & Economics (Operational Research and Statistics Stream)
    - Year 4 of G30C Master of Maths, Op.Res, Stats & Economics (Operational Research and Statistics Stream)
    - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
    - Year 5 of G301 BSc Master of Mathematics, Operational Research, Statistcs and Economics (with Intercalated Year)
    - Year 5 of G30E Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream) Int
    - Year 5 of G30K Master of Maths, Op.Res, Stats & Economics (Data Analysis Stream) Int
    - Year 5 of G30F Master of Maths, Op.Res, Stats & Economics (Econometrics and Mathematical Economics Stream) Int
    - Year 5 of G30G Master of Maths, Op.Res, Stats & Economics (Operational Research and Statistics Stream) Int
    - Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)

This module is Core option list B for:

- UCSA-G4G3 Undergraduate Discrete Mathematics
    - Year 4 of G4G1 Discrete Mathematics
    - Year 4 of G4G3 Discrete Mathematics

This module is Core option list D for:

- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 4 of G30A Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream)
    - Year 4 of G30J Master of Maths, Op.Res, Stats & Economics (Data Analysis Stream)
    - Year 4 of G30B Master of Maths, Op.Res, Stats & Economics (Econometrics and Mathematical Economics Stream)
    - Year 4 of G30C Master of Maths, Op.Res, Stats & Economics (Operational Research and Statistics Stream)
    - Year 4 of G30C Master of Maths, Op.Res, Stats & Economics (Operational Research and Statistics Stream)
    - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)

    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
    - Year 5 of G301 BSc Master of Mathematics, Operational Research, Statistcs and Economics (with Intercalated Year)
    - Year 5 of G30E Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream) Int
    - Year 5 of G30K Master of Maths, Op.Res, Stats & Economics (Data Analysis Stream) Int
    - Year 5 of G30F Master of Maths, Op.Res, Stats & Economics (Econometrics and Mathematical Economics Stream) Int
    - Year 5 of G30G Master of Maths, Op.Res, Stats & Economics (Operational Research and Statistics Stream) Int
    - Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)

This module is Core option list E for:

- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 4 of G30A Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream)
    - Year 4 of G30J Master of Maths, Op.Res, Stats & Economics (Data Analysis Stream)
    - Year 4 of G30B Master of Maths, Op.Res, Stats & Economics (Econometrics and Mathematical Economics Stream)
    - Year 4 of G30C Master of Maths, Op.Res, Stats & Economics (Operational Research and Statistics Stream)
    - Year 4 of G30C Master of Maths, Op.Res, Stats & Economics (Operational Research and Statistics Stream)
    - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
    - Year 5 of G301 BSc Master of Mathematics, Operational Research, Statistcs and Economics (with Intercalated Year)
    - Year 5 of G30E Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream) Int
    - Year 5 of G30K Master of Maths, Op.Res, Stats & Economics (Data Analysis Stream) Int
    - Year 5 of G30F Master of Maths, Op.Res, Stats & Economics (Econometrics and Mathematical Economics Stream) Int
    - Year 5 of G30G Master of Maths, Op.Res, Stats & Economics (Operational Research

and Statistics Stream) Int
- Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)