# ST350-15 Measure Theory for Probability

## 26/27

**Department**
Statistics
**Level**
Undergraduate Level 3
**Module leader**
Elke Thonnes
**Credit value**
15
**Module duration**
10 weeks
**Assessment**
Multiple
**Study location**
University of Warwick main campus, Coventry

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## Description

## Introductory description

Imagine picking a real number x between 0 and 1 "at random" and with perfect accuracy, so that the probability that this number belongs to any interval within [0,1] is equal to the length of the interval. Can we compute the probability of x belonging to any subset to [0,1]?

To answer this question rigorously we need to develop a mathematical framework in which we can model the notion of picking a real number "at random". The mathematics we need, called measure theory, permeates through much of modern mathematics, probability and statistics.

This module provides a strong foundation to the measure theory underpinning probability, concentrating on examples and applications. This module would particularly be useful for students willing to learn more about probability theory, analysis, mathematical finance, and theoretical statistics.

This module is available for students on a course where it is an optional core module or listed option and as an Unusual Option to students who have the required background as specified in the pre-requisite modules.

**Pre-requisites**

- Statistics Students: ST228 Mathematical Methods for Statistics and Probability AND ST229 Probability for Mathematical Statistics AND ST230 Mathematical Statistics
- Non-Statistics Students: ST232/233 Introduction to Mathematical Statistics or ST352 Introduction to Mathematical Statistics (for Finalists)

**Leads to:** ST318 Probability Theory and other advanced probability modules.

For anti-requisite modules please check under the availability tab and in the course handbook.

Module web page

## Module aims

The aims of this module are.

1. Formally and rigorously define measurable spaces.
2. Construct a formal theory of integration with respect to the Lebesgue measure.
3. Apply this formal framework to independence and modes of convergence.
4. Illustrate through examples and application this framework's basis for further studies in probability, statistics and applied mathematics.

## Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Algebras, sigma-algebras and measures
   Algebra and contents, sigma-algebra and measures, pi-systems, examples of random events and measurable sets.
2. Lebesgue integration
   Simple functions, standard representations, measurable functions, Lebesgue integral, properties of integrals, integration of Borel functions.
3. Product measures.
   Sections, product sigma-algebras, product measures, Fubini theorem.
4. Independence and conditional expectation.
   Independence of sigma-algebras, independence of random variables, conditional expectation with respect to a simple algebra.
5. Convergence and modes of convergence
   Borel-Cantelli lemma, Fatou's lemma, dominated convergence theorem, modes of convergence of random variables, Markov's inequality and application, weak and strong laws of large numbers.

## Learning outcomes

By the end of the module, students should be able to:

- Apply properties of the probability spaces to build models for random experiments.
- Evaluate measure-theoretic techniques and apply them to determine probabilities of events.
- Rigorous formulate and apply formal notions of probability, including computing probability, statistical independence and expectation, to a range of situations.
- Apply measure-theoretic integration to a range of situations to derive results regarding random variables from first principles.
- Apply modes of convergence of sequences of random variables to a breath of situations in probability and statistics.
- Apply and justify convergence in the computation of integrals and expectations.

## Indicative reading list

Reading lists can be found in Talis

Specific reading list for the module

## Subject specific skills

- Demonstrate facility with rigorousprobabilistic methods.
- Evaluate, select and apply appropriate mathematical and/or probabilist techniques.
- Demonstrate knowledge of and facility with formal probability concepts, both explicitly and by applying them to the solution of mathematical problems.
- Create structured and coherent arguments communicating them in written form.
- Construct logical mathematical arguments with clear identification of assumptions and conclusions.
- Reason critically, carefully, and logically and derive (prove) mathematical results.

## Transferable skills

- Problem solving: Use rational and logical reasoning to deduce appropriate and well-reasoned conclusions. Retain an open mind, optimistic of finding solutions, thinking laterally and creatively to look beyond the obvious. Know how to learn from failure.
- Self awareness: Reflect on learning, seeking feedback on and evaluating personal practices, strengths and opportunities for personal growth.
- Communication: Written: Present arguments, knowledge and ideas, in a range of formats.
- Professionalism: Prepared to operate autonomously. Aware of how to be efficient and resilient. Manage priorities and time. Self-motivated, setting and achieving goals, prioritising

tasks.

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## Study

## Study time

| Type | Required |
|---|---|
| Lectures | 30 sessions of 1 hour (20%) |
| Tutorials | 5 sessions of 1 hour (3%) |
| Private study | 100 hours (67%) |
| Assessment | 15 hours (10%) |
| Total | 150 hours |

## Private study description

Weekly revision of lecture notes and materials, wider reading, practice exercises and preparing for examination.

## Costs

No further costs have been identified for this module.

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## Assessment

You must pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

### Assessment group B1

| | Weighting | Study time | Eligible for self-certification |
|---|---|---|---|
| Centrally-timetabled examination (On-campus) | 100% | 15 hours | No |

The examination paper will contain four questions, of which the best marks of THREE questions will be used to calculate your grade.

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- Answerbook Pink (12 page)

### Assessment group R1

| | Weighting | Study time | Eligible for self-certification |
|---|---|---|---|
| In-person Examination - Resit | 100% | | No |

The examination paper will contain four questions, of which the best marks of THREE questions will be used to calculate your grade.

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- Answerbook Pink (12 page)

### Feedback on assessment

Solutions and cohort level feedback will be provided for the examination.

Past exam papers for ST350

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## Availability

### Anti-requisite modules

If you take this module, you cannot also take:

- MA359-15 Measure Theory
- ST342-15 Mathematics of Random Events

## Courses

This module is Core optional for:

- Year 3 of USTA-G304 Undergraduate Data Science (MSci)
- Year 3 of USTA-G305 Undergraduate Data Science (MSci) (with Intercalated Year)
- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
- Year 3 of USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 3 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Optional for:

- USTA-G302 Undergraduate Data Science
    - Year 3 of G302 Data Science
    - Year 3 of G302 Data Science
- Year 3 of USTA-G303 Undergraduate Data Science (with Intercalated Year)
- UCSA-G4G1 Undergraduate Discrete Mathematics
    - Year 3 of G4G1 Discrete Mathematics
    - Year 3 of G4G1 Discrete Mathematics
- UCSA-G4G3 Undergraduate Discrete Mathematics
    - Year 3 of G4G1 Discrete Mathematics
    - Year 3 of G4G3 Discrete Mathematics
- Year 3 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 3 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
    - Year 3 of GG14 Mathematics and Statistics
    - Year 3 of GG14 Mathematics and Statistics
- Year 3 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
    - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
    - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
- Year 3 of USTA-Y603 Undergraduate Mathematics,Operational Research,Statistics,Economics (with Intercalated Year)