PX448-10 Mathematical Methods for Physicists III

26/27

Department
: Physics

Level
: Undergraduate Level 4

Module leader
: Katarzyna Macieszczak

Credit value
: 10

Module duration
: 10 weeks

Assessment
: 100% exam

Study location
: University of Warwick main campus, Coventry

Description

Introductory description

One third of this module is on the calculus of variations and two thirds on complex variables. The calculus of variations is concerned with the minimisation of integrals over sets of differentiable functions. Such integrals crop up in many contexts. For example, the ground state wavefunction of a quantum system minimises the expectation value of the energy. The classical equations of motion for both particles and fields can often be obtained by minimising what is called the action functional.

Requiring functions of complex variables to be analytic (differentiable with respect to their complex argument in some domain) turns out to constrain such functions very strongly. As the module shows: only the constant function is differentiable everywhere, analytic functions are actually equal to their Taylor series and not just approximated by them, a function that is once differentiable is differentiable infinitely many times. Complex differentiable functions are clean, they are fun and they are important in physics. For example, response functions like the dielectric response function are analytic functions with the domain, in which the function is analytic, being related to causality.

[Module web page]

Module aims

To help students develop mathematical skills and to cover material needed in 4th year physics modules

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Calculus of Variations:
The idea of a functional and minimization on sets of differentiable functions. Derivation of the Euler-Lagrange equations. Extension to problems with constraints using Lagrange multipliers. Applications to shortest time (path of light rays, Fermat’s principle), shortest length and areas in geometry, mechanics (Hamilton’s principle); variational principle for the lowest eigenvalue

Functions of Complex Variables:
Revision of complex numbers. Functions of complex variables. Complex differentiability, chain rule, product rule. Analytic functions. Cauchy-Riemann equations, solutions to Laplace’s equation. Examples of non-analytic functions.

Contour Integration, Power Series and Calculus of Residues:
Idea of a contour. Statement and derivation of Cauchy’s theorem via Stokes’ theorem. Cauchy’s integral formula and extension to derivatives. Liouville’s theorem. Taylor’s theorem. AFs equal to their Taylor expansions. Classification of zeros and singularities. Existence of Laurent series at isolated singularity. Branch points. Definition of residue. Statement of residue theorem, application to real integrals. Jordan’s lemma and applications to Fourier integrals, integrals with branch cuts.

Learning outcomes

By the end of the module, students should be able to:

- Set up and solve minimization problems with and without constraints and to derive (and solve in simple cases) the corresponding Euler-Lagrange equations
- Identify an analytic function and classify its singularities
- Establish Cauchy's theorem from the identities of vector calculus
- Use the calculus of residues to evaluate definite integrals

Indicative reading list

Reading lists can be found in Talis

Specific reading list for the module

Subject specific skills

Knowledge of mathematics

Transferable skills

Analytical, communication, problem-solving, self-study

Study

Study time

Type	Required
Lectures	15 sessions of 1 hour (15%)
Seminars	5 sessions of 1 hour (5%)
Private study	80 hours (80%)
Total	100 hours

Private study description

Working through lecture notes, solving problems, wider reading, discussing with others taking the module, revising for exam, practising on past exam papers

Costs

No further costs have been identified for this module.

Assessment

You must pass all assessment components to pass the module.

Assessment group B1

Assessment component	Weighting	Study time	Eligible for self-certification
Centrally-timetabled examination (On-campus) Answer 2 questions	100%		No

	Weighting	Study time	Eligible for self-certification

- Students may use a calculator
- Answerbook Green (8 page)

Reassessment component is the same

Feedback on assessment

Personal tutor, group feedback

Past exam papers for PX448

Availability

Courses

This module is Core for:

- Year 3 of UPXA-F303 Undergraduate Physics (MPhys)
- Year 3 of UPXA-F3FA Undergraduate Physics with Astrophysics (MPhys)

This module is Option list A for:

- Year 3 of UPXA-F300 Undergraduate Physics (BSc)
- Year 3 of UPXA-F303 Undergraduate Physics (MPhys)
- Year 4 of UPXA-F301 Undergraduate Physics (with Intercalated Year)
- Year 3 of UPXA-F3F5 Undergraduate Physics with Astrophysics (BSc)
- Year 3 of UPXA-F3FA Undergraduate Physics with Astrophysics (MPhys)