MA9M6-15 Topics in Algebraic Geometry

26/27

Department
Warwick Mathematics Institute
Level
Research Postgraduate Level
Module leader
Miles Reid
Credit value
15
Module duration
10 weeks
Assessment
100% exam
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Algebraic geometry has deep roots going back to Euler. The modern subject has already been through many generations of increasing sophistication and technical perfection.

The subject studies algebraic varieties, that are described as the geometric locus defined by the vanishing of polynomial equations. That slogan cuts both ways: in some cases a set of simultaneous equations is the starting point, in search of a treatment using geometric ideas. Or alternatively, a geometric construction raises questions in algebra of the best way of expressing it in terms of equations. For example, a geometer may already has an algebraic curves in mind, and raise the question of how many functions there are on it, and what are the algebraic equations they satisfy.

Algebraic geometry is one of the current growth areas of pure mathematics, and it has deep and widespread influences in many other areas of science: in pure mathematics (most obviously) geometry, algebra, combinatorics, number theory; but also in interdisciplinary

context related to different flavours of theoretical physics.

Module aims

The course will take PhD students from a modest background in algebra, geometry and topology, and introduce them to the ideas of algebraic geometry, including the technical background in commutative algebra, scheme theory and cohomology, but also the rich theory of algebraic curves and algebraic surfaces and beyond. The ideas here feature a rich corpus of examples and methods for working with them.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

The lecturer has an extremely large choice of topics, and we expect that over the years, different lecturers will cover widely different topics. We list some currently prominent directions that could be covered, but this is not intended to be comprehensive.

- Ideas of toric geometry
- Quotient of a variety by a finite group
- Cyclic quotients singularities and their resolution
- The resolution of surface singularities
- Divisors and line bundles on curves and higher dimensional varieties. Cartier divisors versus Weil divisors
- The Picard group of line bundles up to linear equivalence
- Locally free sheaves and vector bundles
- Rank 2 vector bundles on a curve and elementary transformations of ruled surfaces
- First notions of coherent sheaves and their cohomology
- First examples of algebraic surfaces
- The degree of a line bundle restricted to an algebraic curve in a variety, and the pairing between Pic S and the homology group H_2(S) or the Néron-Severi group N_1(S).
- Local intersection numbers of curves on a nonsingular surface.
- Bézout's theorem for curves in PP^2.
- Intersection numbers of curves on a projective surface.
- The canonical class K_S and the adjunction formula
- First examples of moduli spaces, esp. Hilbert schemes and the Picard variety as moduli space of line bundles up to isomorphism
- Geometric invariant theory and methods of constructing moduli spaces
- Algebraic stacks

Topics in algebraic surfaces:

- contraction of -1-curve
- rational and ruled surfaces. Tsen's theorem
- Extremal rays and minimal models via Mori theory,

- Birational transformations of surfaces and the Cremona group
- Canonical curves and K3 surfaces
- Elliptic surfaces
- The proof of the classification of surfaces via the classical method of adjunction terminates.
- The classification of surfaces via Mori theory
- Graded ring methods
- Examples of surfaces of general type with small invariants
- The Kodaira-Bombieri theorems on projective embeddings of surfaces of general type
- Basic ideas of Hodge theory
- Moduli and periods of K3 surfaces

Learning outcomes

By the end of the module, students should be able to:

- The successful student will take on board the technical ideas of modern algebraic geometry, including schemes and cohomology. They will acquire facility in handling the technical tools and many of the prominent objects of study in geometry. They will also become aware of several of the recent developments in higher dimensional geometry, including the work of several recent Fields Medallists.

Indicative reading list

Reading lists can be found in Talis

Subject specific skills

Appreciation of ideas from different
categories of geometry and their interaction. For example: (1) the topological and algebraic view of vector bundles. (2) Algebraic and complex analytic approaches to cohomology and the Riemann-Roch. (3) Scheme theoretic synthesis of areas of algebraic number theory and algebraic geometry.

Expertise in higher dimensional geometry, one of the current growth areas in pure mathematics. The ideas of higher dimensional birational has many applications to
representation theory and theoretical aspects of quantum field theory.

Transferable skills

- sourcing research material
- prioritising and summarising relevant information
- absorbing and organizing information
- presentation skills (both oral and written)

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Private study	120 hours (80%)
Total	150 hours

Private study description

Review lectured material.
Review lectured material.
Source and prioritise material for project. Write essay.

Costs

No further costs have been identified for this module.

Assessment

You must pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group B

Assessment component	Weighting	Study time	Eligible for self-certification
Oral examination	100%		No

An oral exam involving a presentation by the student, followed by questions from the panel (2 members of the department)

Reassessment component is the same

Feedback on assessment

Students will receive feedback from the course instructor after the oral exam, to cover also areas

like presentation skills and use of technologies (or blackboard)

Past exam papers for MA9M6

Availability

There is currently no information about the courses for which this module is core or optional.