

MA946-15 Introduction to graduate probability theory

26/27

Department

Warwick Mathematics Institute

Level

Research Postgraduate Level

Module leader

Daniel Ueltschi

Credit value

15

Module duration

10 weeks

Assessment

50% coursework, 50% exam

Study location

University of Warwick main campus, Coventry

Description

Introductory description

The module will provide a basic introduction to graduate probability theory and basic principles and universality of scaling limits

- Review of basic probability theory: limit laws, CLT, Markov processes)
- Brownian motion and potential theory: definition and constructions
- Brownian motion as an example of scaling limits
- Discrete Gaussian Free Field as an example of stochastic processes with spatial index sets
- Understanding of various scaling limits beyond the CLT- scale
- Probability measures with spatial dependency structures
- Deviations of Random Matrices and Geometric Consequences
- Large deviation Theory

Module aims

Basic introduction to graduate probability theory and basic principles and universality of scaling limits

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

The purpose of this module is to provide rigorous training in probability theory for students who plan to specialise in this area or expect probability to feature as an essential tool in their subsequent research. It will also be accessible to students who never got into probability theory beyond core-module level taught in the first year and who are eager to get acquainted with basic probability theory, in particular, the aim is to appeal to but not limited to students working in analysis, dynamical systems, combinatorics & discrete mathematics, and statistical mechanics. To include these two different groups of students and to accommodate their needs and various background the module will cover in the first two weeks a steep learning curve into basic probability theory (see part I below). Secondly, the written assessment, 50 % essay with 16 pages, can be chosen either from a list of basic probability theory (standard textbooks in probability and graduate lecture notes in probability theory) or from a list of high level hot research topics including original research papers and reviews and lecture notes (see below).

Part I: Introduction to probability theory

- Random variables, distributions, and convergence criteria
- Law of large numbers
- The Central Limit Theorem
- Markov processes (random walks in discrete time, scaling limit)

Part II: Introduction to probability theory

The aim will be to develop problem-solving skills together with a deep understanding of the main ideas and techniques in probability theory in the following core areas during the following 5-6 weeks:

- Brownian Motion (definition and construction; Blumenthal's 0-1 Law; Donsker's theorem; law of iterated logarithm; local times; Fokker-Planck equation; Wiener measure; Levy metric; Classical Potential theory).
- Discrete Gaussian Free Field (definition; specifications for spatial dependency structures; random walk representation)
- Large deviation theory (Cramer and Sanov theorem; Varadhan Lemma; Schilder's theorem; basic principles (bridge to variational analysis and PDE theory) and applications)

Part III: Optional topics and overview

The third aim and part of the lecture in the remaining weeks will be to provide an overview of important areas of modern probability. These lectures are more of a seminar, respectively review style, and they are geared to enable the students to obtain basic knowledge and overview in most active research areas of probability. The idea is to choose between one or (two) areas from the following list (the module leader chooses according to the demand and interest):

- Wasserstein gradient flow and large deviation theory
- Random Walks: (discrete heat equation; loop measures; loop-erased random walk; intersections; uniform spanning tree; Schramm-Loewner evolution).
- Poisson and Pure-Jump Markov processes: (random measures; point processes; Cox processes;

randomisation; thinning; Palm measures)

- Random combinatorial structures: (Watson processes; random matrices; random partitions; random graphs).
- (continuous) martingales (filtrations and optional times; Doob's inequality; convergence) or more towards analysis, classical potential theory (harmonic functions; heat kernel; Feynman-Kac; Green functions as occupation densities; capacities).
- Concentration of measures
- Random dynamical systems

Learning outcomes

By the end of the module, students should be able to:

- - Review of basic probability theory: limit laws, CLT, Markov processes) - Brownian motion and potential theory: definition and constructions - Brownian motion as an example of scaling limits - Discrete Gaussian Free Field as an example of stochastic processes with spatial index sets - Understanding of various scaling limits beyond the CLT- scale - Probability measures with spatial dependency structures - Deviations of Random Matrices and Geometric Consequences - Large deviation Theory

Indicative reading list

[Reading lists can be found in Talis](#)

Subject specific skills

Develop a deep understanding and applicability of the following topics

-Basic probability theory: limit laws, CLT, Markov processes)

- Brownian motion and potential theory: definition and constructions
- Brownian motion as an example of scaling limits
- Discrete Gaussian Free Field as an example of stochastic processes with spatial index sets
- Understanding of various scaling limits beyond the CLT- scale
- Probability measures with spatial dependency structures
- Deviations of Random Matrices and Geometric Consequences
- Large deviation Theory

Transferable skills

- sourcing research material
- prioritising and summarising relevant information
- absorbing and organizing information
- presentation skills (both oral and written)

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Private study	120 hours (80%)
Total	150 hours

Private study description

Review lectured material.

Review lectured material.

Source and prioritise material for project. Write essay.

Costs

No further costs have been identified for this module.

Assessment

You must pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group C

	Weighting	Study time	Eligible for self-certification
Assessment component			
Course Project	50%		No
An essay (about 16 pages) on a topic discussed between the lecturer and the student.			
Reassessment component is the same			
Assessment component			
Oral examination	50%		No
An oral exam involving a presentation by the student, followed by questions from the panel (2 members of the department)			

Weighting

Study time

Eligible for self-certification

Reassessment component is the same

Feedback on assessment

A copy of the essay with comments will be returned to the student.

Students will receive feedback from the course instructor after the oral exam, to cover also areas like presentation skills and use of technologies (or blackboard)

[Past exam papers for MA946](#)

Availability

There is currently no information about the courses for which this module is core or optional.