

MA4N6-15 Finite-Element methods for PDEs

26/27

Department

Warwick Mathematics Institute

Level

Undergraduate Level 4

Module leader

Bjorn Stinner

Credit value

15

Module duration

10 weeks

Assessment

Multiple

Study location

University of Warwick main campus, Coventry

Description

Introductory description

The module will introduce students to the analysis and numerical approximation of variational solutions of partial differential equations (PDEs). Many mathematical models are based on PDEs but their complexity generally make it impossible to solve these explicitly. Existence and uniqueness of solutions to these problems can be shown based on their variational formulation but this on the face of it, does not provide a practical way of evaluating solutions. In this module we will discuss how the variational formulation can be used to construct so called Galerkin approximations. The most commonly used form of these is the finite element method. We will discuss both implementational and analytical aspects of these method in this module.

We will start with an overview of variational formulations of PDEs and the relevant function spaces so that MA4A2 (Advanced PDEs) is not a prerequisite. Some results from functional analysis will only be recalled, e.g., Riesz representation and Lax-Milgram theorems.

Module aims

Students will learn about the variational formulation of linear PDEs and how to use to these to construct approximations using Galerkin methods. A focus will be on the Finite-Element method,

discussing both implementational aspects and their a-priori and a-posteriori error analysis.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

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- Sobolev spaces and their main properties in connection with variational formulations of PDEs
- Galerkin approximations of variational problems
- Introduction to implementational aspects of finite element methods
- Approximation theory and a-priori error analysis
- Derivation of a-posteriori error estimators
- How the concepts are used to solve non-linear and time dependent problems

Learning outcomes

By the end of the module, students should be able to:

- have an overview of Sobolev spaces and their main properties
- understand the fundamental concepts of Galerkin approximations to weak solutions of linear PDEs
- implement finite-element methods to compute approximations
- carry out a-priori error analysis based on the Bramble-Hilbert theory
- understand the difference between a-priori and a-posteriori analysis and how to obtain a-posteriori error estimators.
- understand how the Galerkin method can be used to solve non-linear and time dependent problems

Indicative reading list

[Reading lists can be found in Talis](#)

Subject specific skills

After completing this module, students will be able to:

- Recall the definition of the Sobolev spaces and formulate notions of weak solutions for linear elliptic PDEs.
- approximate variational problems using Galerkin methods
- implement finite-element methods
- prove a-priori and a-posteriori error estimates for finite element approximations
- use the finite-element method to also solve non-linear and time dependent PDEs

Transferable skills

Students will have a detailed understanding how PDEs are formulated in variational form and how this formulation can be used to construct approximations. They will have seen both theoretical and practical aspects of one of the most widely used numerical methods for solving PDEs both in academia and industry. They will have obtained a foundation for further study at graduate level, and will be able to apply the techniques they have learned in pure mathematics, the applied sciences and in many branches of industry.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Working on assignments, going over lecture notes, text books, exam revision.

Costs

No further costs have been identified for this module.

Assessment

You must pass all assessment components to pass the module.

Assessment group B

	Weighting	Study time	Eligible for self-certification
Centrally-timetabled examination (On-campus)	100%		No
In person 3 hour exam			

	Weighting	Study time	Eligible for self-certification
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- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
In-person Examination	100%		No

- Answerbook Gold (24 page)

Feedback on assessment

Exam feedback

[Past exam papers for MA4N6](#)

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
 - Year 1 of G1PC Mathematics (Diploma plus MSc)
 - Year 2 of G1PC Mathematics (Diploma plus MSc)

This module is Core option list F for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list B for:

- Year 4 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 5 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)

This module is Option list C for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
 - Year 4 of G105 Mathematics (MMath) with Intercalated Year
 - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G103 Undergraduate Mathematics (MMath)

- Year 3 of G103 Mathematics (MMath)
 - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad