MA4L6-15 Analytic Number Theory

26/27

Department
: Warwick Mathematics Institute

Level
: Undergraduate Level 4

Module leader
: Sam Chow

Credit value
: 15

Module duration
: 10 weeks

Assessment
: Multiple

Study location
: University of Warwick main campus, Coventry

Description

Introductory description

Analytic number theory uses methods of real and complex analysis to study the distribution of interesting number theoretic objects. It is a huge subject, so to focus the course we will concentrate on questions about the distribution of primes, and mostly on those questions that can be attacked using the Riemann zeta function and related objects. This is the subject for which the zeta function and related functions were introduced and studied in the first place (by Euler, Dirichlet, Riemann, ...).

By ``the distribution of primes'', I mean questions like:

- how many primes are less than x ? (for an arbitrary large number x)
- how many primes are less than x and congruent to a modulo q, for given natural numbers a and q (possibly allowed to vary with x)?

Module web page

Module aims

Multiplicative number theory studies the distribution of objects, like prime numbers or numbers with few prime factors or small prime factors, that are multiplicatively defined. A powerful tool for this is the analysis of generating functions like the Riemann zeta function, a method introduced in the 19th century that allowed the resolution of problems dating back to the ancient Greeks. This course will introduce some of these questions and methods.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

The course will cover some of the following topics, depending on time and audience preferences:
(1) Warm-up: The counting functions pi(x), Psi(x) of primes up to x. Chebychev's upper and lower bounds for Psi(x).
(2) Basic theory of the Riemann zeta function: Definition of the zeta function zeta(s) when Re(s) > 1, and then when Re(s) > 0 and for all s. The connection with primes via the Euler product. Proof that zeta(s) neq 0 when Re(s) geq 1, and deduction of the Prime Number Theorem (asymptotic for Psi(x)).
(3) More on zeros of zeta: Non-existence of zeta zeros follows from estimates for sum_{N < n < 2N} n^{it}. The connection with exponential sums, and outline of the methods of Van der Corput and Vinogradov. Wider zero-free regions for zeta(s), and application to improving the Prime Number Theorem. Statement of the Riemann Hypothesis.
(4) Primes in arithmetic progressions: Dirichlet characters chi and Dirichlet L-functions

Learning outcomes

By the end of the module, students should be able to:

- Have a good understanding of the Riemann zeta function and the theory surrounding it up to the Prime Number Theorem.
- Understand and appreciate the connection of the zeros of the zeta function with exponential sums and the statement of the Riemann Hypothesis.
- Demonstrate the necessary grasp and understanding of the material to potentially pursue further postgraduate study in the area.
- Consolidate existing knowledge from real and complex analysis and be able to place in the context of Analytic Number Theory

Indicative reading list

Reading lists can be found in Talis

Subject specific skills

By the end of the module the student should be able to:

- consolidate existing knowledge from real and complex analysis in the context of Analytic Number Theory.
- use real variable methods such as those of Chebychev and Mertens, as well as summation by parts, to estimate certain sums over prime numbers.

- understand the Riemann zeta function and the theory surrounding it, including using the zeta function to prove the Prime Number Theorem.
- understand and appreciate the connection of the zeros of the zeta function with exponential sums and the statement of the Riemann Hypothesis.
- potentially pursue further postgraduate study in the area.

Transferable skills

The module will help to develop skills in understanding, assessing and constructing sophisticated logical arguments (especially of a quantitative nature), and presenting these clearly in writing. In the optional support classes, students will also have opportunities to work collectively and to present their arguments orally.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	5 sessions of 1 hour (3%)
Private study	115 hours (77%)
Total	150 hours

Private study description

5 hours of support classes, once per fortnight during the term. 115 hours to review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group B1

	Weighting	Study time	Eligible for self-certification
Centrally-timetabled examination (On-campus)	100%		No

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

- Answerbook Gold (24 page)

Feedback on assessment

Coursework discussed collectively in the support classes and model solutions are provided. Exam feedback.

[Past exam papers for MA4L6]

Availability

Courses

This module is Optional for:

- TMAA-G1PE Master of Advanced Study in Mathematical Sciences
  - Year 1 of G1PE Master of Advanced Study in Mathematical Sciences
  - Year 1 of G1PE Master of Advanced Study in Mathematical Sciences
- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
  - Year 1 of G1PC Mathematics (Diploma plus MSc)
  - Year 2 of G1PC Mathematics (Diploma plus MSc)

This module is Core option list F for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
    - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
    - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Option list B for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
    - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
    - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- Year 4 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 5 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)

This module is Option list C for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
    - Year 3 of G105 Mathematics (MMath) with Intercalated Year
    - Year 4 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
    - Year 3 of G106 Mathematics (MMath) with Study in Europe
    - Year 4 of G106 Mathematics (MMath) with Study in Europe