MA4G6-15 Calculus of Variations

26/27

Department
    Warwick Mathematics Institute
Level
    Undergraduate Level 4
Module leader
    Filip Rindler
Credit value
    15
Module duration
    10 weeks
Assessment
    Multiple
Study location
    University of Warwick main campus, Coventry

Description

Introductory description

The module provides a modern introduction to the calculus of variations, focusing on finding the extrema (minima or maxima) of functionals—mappings from functions to real numbers. It moves beyond classical techniques to explore powerful methods used to investigate nonlinear problems in physics, materials science, and geometry.

Module aims

The Calculus of Variations is both old and new; starting from Euler's work up to very recent discoveries, this sub-field of Mathematical Analysis has proven to be very successful in the analysis of physical, technological and economic systems. This is due to the fact that many such systems incorporate some kind of variational (minimum, maximum, extremum) principle and understanding this structure is paramount to proving meaningful results about them. Applications range from material sciences to geometry to optimal control theory. The aim of this course is to give a thoroughly modern introduction and to lead from the basics to sophisticated recent results.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Various models leading to variational problems.

Sobolev spaces.

The Direct Method of the Calculus of Variations and lower semicontinuity.

Convexity and aspects of Convex Analysis (duality).

Existence of solutions for scalar problems.

Polyconvexity and existence of solutions semicontinuity for vector-valued problems.

Regularity theory for minimisation problems.

Optimal control theory and Young measures.

Quasiconvexity, laminates and microstructure.

Variational convergence of functionals (.-convergence).

If time permits: Other variational principles (Ekeland etc.).

If time permits: Functions of bounded variations and applications.

Learning outcomes

By the end of the module, students should be able to:

- Understand why variational problems are important.
- See several examples of variational problems in physics and other sciences.
- Appreciate that (and why) some problems have “classical” solutions and some do not.
- Be able to prove the existence of solutions to convex variational problems.
- Know which kinds of problems are not convex and why convexity is often an unrealistic assumption for vector-valued problems.
- Have an insight into generalised convexity conditions, such a quasiconvexity and polyconvexity and their applications.
- Be able to prove existence of solutions to quasiconvex/polyconvex variational problems.
- Have seen simple optimal control problems and can understand them as a special case of general variational problems.
- Know what microstructure is, why it forms, and what its physical significance is.
- Have seen how regularised functionals converge to a limit functional as the regularisation parameter tends to zero.

Indicative reading list

Reading lists can be found in Talis

Subject specific skills

At the conclusion of the module the student should be able to set up and solve various minimisation problems with and without constraints, to derive Euler-Lagrange equations and appreciate how the laws of mechanics and geometrical optics, as well as some geometrical problems involving least length and least area, fit into this framework.

Transferable skills

1. Precise Formulation: The capacity to restate fundamental laws of physics into precise mathematical frameworks.
2. Abstract Reasoning: Acquiring a mature understanding of abstract mathematical ideas and their real-world applications.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Homework, assignments, engagement with departmental support and feedback mechanisms, exam preparation.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group B1

	Weighting	Study time	Eligible for self-certification
In-person Examination	100%		No
3 hour exam, no books allowed.			

	Weighting	Study time	Eligible for self-certification

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
Centrally-timetabled examination (on-campus)	100%		No

Feedback on assessment

Solutions to homework problems, support classes, solutions to exam papers.

[Past exam papers for MA4G6]

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
  - Year 1 of G1PC Mathematics (Diploma plus MSc)
  - Year 2 of G1PC Mathematics (Diploma plus MSc)

This module is Option list A for:

- Year 4 of UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)

This module is Option list C for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad