MA467-15 Presentations of Groups

26/27

Department
: Warwick Mathematics Institute

Level
: Undergraduate Level 4

Module leader
: Derek Holt

Credit value
: 15

Module duration
: 10 weeks

Assessment
: Multiple

Study location
: University of Warwick main campus, Coventry

Description

Introductory description

This module is about groups that are defined by means of a presentation in terms of generators and relations. This means that a set of generators X is given for the group G, and a set of defining relations R. Defining relations are equations involving the generators and their inverses, which are required to hold in G. Then G is defined to be essentially the largest group that is generated by a set X for which the defining relations hold. For example, the dihedral group of order 6 could be defined as the group with generating set X = {x,y} and relations R = {x3=1, y2=1, yxy=x-1}.

This method of defining a group has the advantage that it is often the most concise description of the group possible. Furthermore, groups arising from algebraic topology often appear naturally in this form. The disadvantage of the method is that it can be very difficult (and even theoretically impossible in some cases) to derive important properties of a group G that is given only by a presentation, such as whether it is finite, abelian, etc., However, as a result of the frequency with which group presentations crop up in other branches of mathematics, the development of techniques for finding out information about these groups has become a major branch of mathematical research.
In this module, we shall be developing the basic theory of group presentations, and looking at some particular techniques for analysing them. We start with free groups (groups with no defining relations) and prove a fundamental theorem of Schreier, that a subgroup of a free group is itself free. We then move on to presentations in general, and look at lots of examples. In the later part of

the module, we shall be looking at some algorithmic methods for studying group presentations, including the Todd-Coxeter algorithm for calculating the index of a subgroup H of finite index in G, and the Reidemeister-Schreier method for calculating a presentation of H. (These algorithms are highly suitable for computer implementation, although we will not be studying that aspect of them in detail in this course.)

Module aims

To illustrate the important general notion of definition of an algebraic structure by generators and defining relations in the context of group theory.

To develop some examples of the use of algorithmic methods in pure mathematics.

To give a mathematically precise but comprehensible treatment of the definition of a group by generators and relations, and to learn how to start extracting elementary information about the group from its presentation.

To learn how to carry out the Todd-Coxeter coset enumeration algorithm by hand in simple examples, and how to compute presentations of subgroups of groups.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Revision of idea of generators of a group.
2. Basic properties of free groups.
3. Algebraic proof that a subgroup of a free group is free.
4. Definition, basic properties, and examples of group presentations.
5. The Todd-Coxeter coset enumeration algorithm with examples and correctrness proof.
6. Presentations of subgroups and the Reidemeister-Schreier algorithm for computing them.
7. The Baumslag-Solitar groups and the non-Hopfian examples.
8. The Burnside Problem and the Grigorchuk group.

Learning outcomes

By the end of the module, students should be able to:

- Understand basic theory of group presentations.
- Extracting elementary information about the group from its presentation.
- Ability to carry out the Todd-Coxeter coset enumeration algorithm by hand in simple examples, and how to compute presentations of subgroups of groups.
- Computing the index of a subgroup of finite index in a group defined by a presentation.

Indicative reading list

Reading lists can be found in Talis

Subject specific skills

Problem solving, calculation and experiment leading to clarity and proof. The importance of systematic and symmetrical organization. The complementarity of example and abstraction. Formulating problems as algorithms, thereby enhancing understanding of details and rendering them suitable for computer implementation.

Transferable skills

Clear and precise thinking, and the ability to follow complex reasoning, to construct logical arguments, and to expose illogical ones. The ability to retrieve the essential details from a complex situation and thereby facilitate problem resolution.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	9 sessions of 1 hour (6%)
Tutorials	(0%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Homework, assignments, engagement with departmental support and feedback mechanisms, exam preparation.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group B2

	Weighting	Study time	Eligible for self-certification
In-person Examination	100%		No

3 hour exam, no books allowed

Assessment group R1

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

- Answerbook Gold (24 page)

Feedback on assessment

Solutions to homework problems, support classes, solutions to exam paper.

Past exam papers for MA467

Availability

Courses

This module is Option list A for:

- Year 4 of UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)

This module is Option list C for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)