MA3L2-15 Optimisation

26/27

Department
Warwick Mathematics Institute
Level
Undergraduate Level 3
Module leader
Susana Gomes
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

The solution of optimisation problems is at the core of several applications, from decision-making to engineering design problems. In these contexts, one often wants to optimise an outcome (e.g. minimise the cost of production) of a process, where the process variables are restricted to certain constraints. In today's data-driven world, one would also want to find the best fit between existing models and available data. Often, both costs and constraints are nonlinear functions of the existing variables. This module will introduce key concepts in the mathematical analysis of (continuous) optimisation problems, starting with classical methods and their convergence properties, both for unconstrained and constrained problems, including derivative-free approaches when gradients are not available, and finishing with more modern methods for the solution of these nonlinear problems, especially when they have a large number of variables.

Module web page

Module aims

This module aims to introduce mathematics students to continuous optimisation problems as well as their solution, when possible, or common bottlenecks, and how to tackle them when a solution does not exist. It will focus on several classes of optimisation methods and use both analytical (e.g. convergence properties) and numerical approaches to study them, finishing with an overview of modern research problems in the field.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Motivation. Why we need optimisation, how to recognise solutions (basic notions of critical points and their classification) and quantify convergence, what can go wrong, why we need sophisticated algorithms (large data, non-smooth functions, local vs global minima)
2. Introduction to unconstrained optimisation. Optimality conditions. Choose from: line search methods, steepest descent methods, Newton and quasi-Newton methods, trust region methods, conjugate gradients.
   Convergence rates. Global vs local optimisation / Convex vs non-convex problems
3. Derivative-free optimisation (time permitting). Basic notions of convex sets and functions, tangent cones, cone of linearised feasible directions. Examples of algorithms (e.g. Nelder-Mead and other recent developments)
4. Constrained optimisation. types of and qualification of restrictions, necessary and sufficient conditions (first and second order, optimality/KKT conditions), duality theory, linear programming, quadratic programming (SQP) Farkas lemma.
5. Numerical methods for constrained optimisation. Quadratic penalisation, augmented Lagrangian, interior point / barrier methods
6. Extra topics. Choose from: Particle Swarm Optimisation, Simulated Annealing, Consensus Based Optimisation, Stochastic Gradient Descent

Learning outcomes

By the end of the module, students should be able to:

- - Understand critical points of multivariable functions.
- - Understand convex and nonconvex problems and associated difficulties in the context of optimisation.
- - Apply several techniques to solve nonlinear / nonconvex optimisation problems.
- - Derive convergence rates or guarantees for a number of optimisation algorithms.
- - Compare different optimisation techniques and choose the appropriate technique for example problems.
- - Apply appropriate numerical methods to solve optimisation problems.

Indicative reading list

Reading lists can be found in Talis

Specific reading list for the module

Subject specific skills

Students should be able to translate a real-world problem into an optimisation problem and its constraints. Identify convexity or nonconvexity of a problem and the appropriate method to solve them. They should also be able to discuss the difficulties appearing for nonconvex or nonsmooth problems, this motivating derivative-free optimisation. Compare and contrast different methods. Solve these problems numerically for given examples.

Transferable skills

Critical thinking and discussion, application of mathematics to real-world problems, handling large data sets. Ability to translate real-world ideas into mathematical language. Programming.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Homework, assignments, engagement with departmental support and feedback mechanisms, exam preparation.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D1

	Weighting	Study time	Eligible for self-certification
Assignments	15%		No
Assignments (including theoretical and (small) programming exercises with code provided)			
Centrally-timetabled examination (On-	85%		No

	Weighting	Study time	Eligible for self-certification
campus) Examination			

Assessment group R1

	Weighting	Study time	Eligible for self-certification
In-person Examination	100%		No
Exam			

Feedback on assessment

Solutions to homework problems, support classes, solutions to exam papers. Availability of code to compare results from.

[Past exam papers for MA3L2]

Availability

Courses

This module is Optional for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G100 Mathematics
  - Year 4 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year