

MA3K6-15 Boolean Functions

26/27

Department

Warwick Mathematics Institute

Level

Undergraduate Level 3

Module leader

Vadim Lozin

Credit value

15

Module duration

10 weeks

Assessment

Multiple

Study location

University of Warwick main campus, Coventry

Description

Introductory description

Boolean functions, named after an English mathematician George Boole, are $\{0,1\}$ -valued functions of $\{0,1\}$ -valued variables. They are the most fundamental objects studied in pure and applied mathematics. This module is a comprehensive introduction to the theory of Boolean functions with an emphasis on structural and combinatorial properties. We will also discuss connections of Boolean functions with other combinatorial structures, such as graphs, hypergraphs, set systems, matroids, as well as applications of Boolean functions in data analysis and optimisation.

Module aims

The module will include a brief introduction to the basic concepts of Boolean functions, and it will then be structured around the following topics:

1. Foundations

- Representations of Boolean functions: Boolean expressions, binary decision diagrams, decision trees, geometric interpretation
- Normal forms, complexity measures
- Duality theory

- Functional completeness, Post's Theorem

1. Classes of Boolean functions

- Quadratic functions
- Horn functions
- Threshold functions
- Read-once functions

1. Generalizations and applications

- Partially defined Boolean functions and logical analysis of data
- Pseudo-Boolean functions and pseudo-Boolean optimisation

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

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Learning outcomes

By the end of the module, students should be able to:

- 1. Understand the notion of Boolean function and various ways of representing them
- 2. State Post's Theorem and compute Post classes generated by given functions
- 3. Recognize special classes of Boolean functions, such as quadratic, Horn, threshold, read-once, etc.
- 4. Be able to apply partially defined Boolean functions to logical analysis of data
- 5. Understand pseudo-Boolean functions in the context of pseudo-Boolean optimisation

Indicative reading list

[Reading lists can be found in Talis](#)

Subject specific skills

This module lays a solid foundation to the study of Boolean functions. It develops in depth the notions that allow one to investigate the structure of Boolean functions and apply them to Algebra, Analysis, Combinatorics

or Computer Science. Students taking the module will learn some of the techniques required for working on a large-scale research project. These techniques are partly theoretical, and partly computational.

Transferable skills

Clear and precise thinking, and the ability to follow complex reasoning, to construct logical arguments, and to expose illogical ones. The ability to retrieve the essential details from a complex situation and thereby facilitate problem resolution.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (29%)
Tutorials	9 sessions of 1 hour (9%)
Online learning (independent)	(0%)
Private study	63 hours (62%)
Total	102 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time	Eligible for self-certification
Homeworks	15%	15 hours	No
Centrally-timetabled examination (On-campus)	85%	33 hours	No

3 hour exam, no books allowed

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- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

- Answerbook Gold (24 page)

Feedback on assessment

Exam and marked Homeworks feedback

[Past exam papers for MA3K6](#)

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
 - Year 1 of G1PC Mathematics (Diploma plus MSc)
 - Year 2 of G1PC Mathematics (Diploma plus MSc)
- Year 3 of UCSA-G4G1 Undergraduate Discrete Mathematics
- UCSA-G4G3 Undergraduate Discrete Mathematics
 - Year 3 of G4G1 Discrete Mathematics
 - Year 3 of G4G3 Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year

This module is Core option list A for:

- Year 4 of UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year

This module is Core option list B for:

- Year 3 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list C for:

- Year 3 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- Year 4 of UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list F for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 2 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
 - Year 4 of G105 Mathematics (MMath) with Intercalated Year
 - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- Year 3 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
 - Year 3 of G100 Mathematics
 - Year 3 of G103 Mathematics (MMath)
 - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad
- Year 4 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- Year 3 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
 - Year 3 of GF13 Mathematics and Physics
 - Year 3 of FG31 Mathematics and Physics (MMathPhys)
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year

This module is Option list B for:

- Year 3 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)

This module is Option list C for:

- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)