MA3H6-15 Algebraic Topology

26/27

Department
Warwick Mathematics Institute
Level
Undergraduate Level 3
Module leader
Martin Gallauer
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Algebraic topology is concerned with the construction of algebraic invariants (usually groups) associated to topological spaces which serve to distinguish between them. Most of these invariants are ``homotopy'' invariants. In essence, this means that they do not change under continuous deformation of the space and homotopy is a precise way of formulating the idea of continuous deformation. This module will concentrate on constructing the most basic family of such invariants, homology groups, and the applications of these homology groups.

Module web page

Module aims

To introduce homology groups for simplicial complexes; to extend these to the singular homology groups of topological spaces; to prove the topological and homotopy invariance of homology; to give applications to some classical topological problems.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

The starting point will be simplicial complexes and simplicial homology. An n-simplex is the n-dimensional generalisation of a triangle in the plane. A simplicial complex is a topological space which can be decomposed as a union of simplices. The simplicial homology depends on the way these simplices fit together to form the given space. Roughly speaking, it measures the number of p-dimensional "holes" in the simplicial complex. For example, a hollow 2-sphere has one 2-dimensional hole, and no 1-dimensional holes. A hollow torus has one 2-dimensional hole and two 1-dimensional holes. Singular homology is the generalisation of simplicial homology to arbitrary topological spaces. The key idea is to replace a simplex in a simplicial complex by a continuous map from a standard simplex into the topological space. It is not that hard to prove that singular homology is a homotopy invariant but very hard to compute singular homology directly from the definition. One of the main results in the module will be the proof that simplicial homology and singular homology agree for simplicial complexes. This result means that we can combine the theoretical power of singular homology and the computability of simplicial homology to get many applications. These applications will include the Brouwer fixed point theorem, the Lefschetz fixed point theorem and applications to the study of vector fields on spheres.

Learning outcomes

By the end of the module, students should be able to:

- Give the definitions of simplicial complexes and their homology groups and a geometric understanding of what these groups measure
- Give techniques for computing these groups
- Give the extension to singular homology
- Understand the theoretical power of singular homology
- Develop a geometric understanding of how to use these groups in practice

Indicative reading list

Reading lists can be found in Talis

Subject specific skills

By the end of the module the student should be able to develop a geometric understanding of how to use these groups in practice

Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	10 sessions of 1 hour (7%)
Private study	110 hours (73%)
Total	150 hours

Private study description

110 hours: private study, assignment sheets, revision for exam.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group D1

	Weighting	Study time	Eligible for self-certification
Assignment Sheets Assignment sheets.	15%		No
Centrally-timetabled examination (On-campus) A 3-hour written exam.	85%		No

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- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

- Answerbook Gold (24 page)

Feedback on assessment

Support classes, work returned after marking and exam feedback.

Past exam papers for MA3H6

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
    - Year 1 of G1PC Mathematics (Diploma plus MSc)
    - Year 2 of G1PC Mathematics (Diploma plus MSc)
- Year 3 of UCSA-G4G1 Undergraduate Discrete Mathematics
- UCSA-G4G3 Undergraduate Discrete Mathematics
    - Year 3 of G4G1 Discrete Mathematics
    - Year 3 of G4G3 Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- Year 3 of UMAA-GL11 Undergraduate Mathematics and Economics

This module is Core option list A for:

- Year 4 of UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year

This module is Core option list B for:

- Year 3 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list C for:

- Year 3 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- Year 4 of UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list F for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 3 of G105 Mathematics (MMath) with Intercalated Year
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- Year 3 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
  - Year 4 of G106 Mathematics (MMath) with Study in Europe
- Year 3 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 3 of GF13 Mathematics and Physics
  - Year 3 of FG31 Mathematics and Physics (MMathPhys)
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

This module is Option list C for:

- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Option list E for:

- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and

Economics (with Intercalated

- Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)