

# MA3G6-15 Commutative Algebra

26/27

**Department**

Warwick Mathematics Institute

**Level**

Undergraduate Level 3

**Module leader**

Marco Schlichting

**Credit value**

15

**Module duration**

10 weeks

**Assessment**

Multiple

**Study location**

University of Warwick main campus, Coventry

---

## Description

### Introductory description

Commutative Algebra is the study of commutative rings, and their modules and ideals. This theory has developed over the last 150 years not just as an area of algebra considered for its own sake, but as a tool in the study of two enormously important branches of mathematics: algebraic geometry and algebraic number theory. The unification which results, where the same underlying algebraic structures arise both in geometry and in number theory, has been one of the crowning glories of twentieth century mathematics and still plays an absolutely fundamental role in current work in both these fields.

One simple example of this unification will be familiar already to anyone who has noticed the strong parallels between the ring  $\mathbb{Z}$  (a Euclidean Domain and hence also a Unique Factorization Domain) and the ring  $F[X]$  of polynomials over a field (which has both the same properties). More generally, the rings of algebraic integers which have been studied since the 19th century to solve problems in number theory have parallels in rings of functions on curves in geometry.

While self-contained, this course will also serve as a useful introduction to either algebraic geometry or algebraic number theory.

[Module web page](#)

### Module aims

Commutative Algebra is the study of commutative rings, and their modules and ideals. The module Commutative Algebra is to build bridges connecting algebra, geometry, and number theory. Also it serves as a preliminary module for both algebraic geometry and algebraic number theory.

## Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Topics: Gröbner bases, modules, localization, integral closure, primary decomposition, valuations and dimension.

## Learning outcomes

By the end of the module, students should be able to:

- understand the Hilbert bases theorem, know several examples of Noetherian rings.
- be able to compute the Gröbner bases of an ideal of the polynomial ring; Spec of some rings; zero locus of ideals.
- understand the proof of Cayley-Hamilton theorem, and to be able to apply it to prove Nakayama Lemma, Tower's law and other theorems in module theory.
- be able to determine algebraic integers via several methods in the theory of finite extensions.
- be able to compute the localisation, integral closure, dimension of rings.
- understand the proof of Hilbert Nullstellensatz, and to be able to apply it to compute the primary decomposition of ideals in a polynomial ring.

## Indicative reading list

[Reading lists can be found in Talis](#)

## Subject specific skills

The students will get their first knowledge in algebraic geometry and algebraic number theory. The students will train their programming skill in various assignments, especially computational problems for Gröbner bases.

## Transferable skills

The students will acquire confidence in dealing with apparently complicated problems which nonetheless have a simple underlying solution.

---

## Study

## Study time

| Type          | Required                    |
|---------------|-----------------------------|
| Lectures      | 30 sessions of 1 hour (20%) |
| Tutorials     | 9 sessions of 1 hour (6%)   |
| Private study | 111 hours (74%)             |
| Total         | 150 hours                   |

## Private study description

Review lectured material and work on set exercises.

## Costs

No further costs have been identified for this module.

---

## Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

### Assessment group D1

|                                              | Weighting | Study time | Eligible for self-certification |
|----------------------------------------------|-----------|------------|---------------------------------|
| Coursework                                   | 15%       |            | No                              |
| Centrally-timetabled examination (On-campus) | 85%       |            | No                              |

- Answerbook Gold (24 page)

### Assessment group R

|                               | Weighting | Study time | Eligible for self-certification |
|-------------------------------|-----------|------------|---------------------------------|
| In-person Examination - Resit | 100%      |            | No                              |

- Answerbook Gold (24 page)

## Feedback on assessment

Marked coursework and exam feedback.

[Past exam papers for MA3G6](#)

---

## Availability

### Courses

This module is Optional for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
  - Year 1 of G1PC Mathematics (Diploma plus MSc)
  - Year 2 of G1PC Mathematics (Diploma plus MSc)
- Year 3 of UCSA-G4G1 Undergraduate Discrete Mathematics
- UCSA-G4G3 Undergraduate Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
  - Year 3 of G4G3 Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- Year 3 of UMAA-GL11 Undergraduate Mathematics and Economics

This module is Core option list A for:

- Year 4 of UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year

This module is Core option list B for:

- Year 3 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list C for:

- Year 3 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- Year 4 of UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list F for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 3 of G105 Mathematics (MMath) with Intercalated Year
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- Year 3 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
  - Year 4 of G106 Mathematics (MMath) with Study in Europe
- Year 3 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 3 of GF13 Mathematics and Physics
  - Year 3 of FG31 Mathematics and Physics (MMathPhys)
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

This module is Option list C for:

- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Option list E for:

- USTA-G301 Undergraduate Master of Mathematics, Operational Research, Statistics and

### Economics (with Intercalated

- Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)