MA3B4-15 Probability: Structures and Examples

26/27

Department
Warwick Mathematics Institute
Level
Undergraduate Level 3
Module leader
Nikolaos Zygouras
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

This is a new second year module that is intended as a bridge between first year 'introduction to probability' type modules and third year probability modules. It emphasises the practical application of probabilistic theorems in particular examples.

Module aims

To introduce students to the main limit theorems and probability inequalities that apply to models based on large numbers of independent random variables.

To show applications of these results to concrete questions about a wide range of such models, drawing from graph theory, coding theory, random walks, statistical mechanics and possibly other fields of modern probability trends or where randomness appears in fields of pure maths.

To expose students to deeper and more rigorous arguments in probability that are based on analysis but are not dependent on MA359 Measure Theory and also start developing a probabilistic understanding and intuition.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Recap of basic probability (independence, expectations, variance, correlation and correlation inequalities) and motivating examples
2. The weak law of large numbers
3. Notions of convergence in probability
4. Borel—Cantelli lemma and strong law of large numbers
5. Simple large deviations estimates
6. Shannon entropy and some applications to combinatorics and coding theory
7. The central limit theorem (CLT): proof of the de Moivre-Laplace CLT via by-hand computation and Stirling formula and introduction to the Lindeberg exchange principle
8. Poisson random variables and convergence
9. Examples. This will constitute a large part of the module, to demonstrate both the applications of the above theoretical principles and to develop computational skills. Examples will be drawn from combinatorial probability (random graphs, random permutations), stochastic processes (e.g. random walks, Gambler's ruin) statistical mechanics, information theory

Learning outcomes

By the end of the module, students should be able to:

- Familiarise with the notions of probabilistic convergence
- Main probabilistic inequalities and their applications to functions of long strings of i.i.d. random variables, giving proofs of the weak law of large numbers under various hypothesis and also associated large-deviations inequalities
- Apply those results to concrete models in combinatorics, coding theory, statistical mechanics, simple cases of random walks and possibly examples from other fields of pure maths (permutations and groups or analysis) where randomness is present or probabilistic tools become handy.
- State the central limit theorem, and understand the main ideas in one of its proofs for finite-valued random variables.
- State Poisson convergence and highlight the dichotomy between central limit behaviour vs Poisson.

Subject specific skills

Understand the basic methods of rigorous probability such as the weak law of large numbers and the central limit theorem, and be able to apply these techniques in examples.

Transferable skills

Understand some of the applications of the techniques in this module to coding theory, combinatorics and statistical mechanics.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Homework, assignments, engagement with departmental support and feedback mechanisms, exam preparation.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group B

	Weighting	Study time	Eligible for self-certification
Centrally-timetabled examination (on-campus)	100%		No

Assessment group R

	Weighting	Study time	Eligible for self-certification
Centrally-timetabled examination (on-campus)	100%		No

Feedback on assessment

Solutions to homework problems, support classes.

[Past exam papers for MA3B4](#)

Availability

Courses

This module is Option list A for:

- Year 4 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- Year 3 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year