

MA357-10 Introduction to Number Theory

26/27

Department

Warwick Mathematics Institute

Level

Undergraduate Level 3

Module leader

Samir Siksek

Credit value

10

Module duration

10 weeks

Assessment

Multiple

Study location

University of Warwick main campus, Coventry

Description

Introductory description

Number theory is an ancient and beautiful subject that investigates the properties of the integers, and other questions motivated by them. For example, one studies questions like:

- do equations have integer solutions? If so, do they have infinitely many integer solutions? How are those solutions distributed, and can we parametrise them? Pythagorean triples, and Fermat's Last Theorem, fall into this general area of Diophantine equations.
- how do special subsets of the integers behave? The primes are probably the best example of a special subset, and one wants to know: are there infinitely many? How are they distributed? Are there interesting patterns amongst them? The most famous unsolved problem in mathematics, the Riemann Hypothesis, is connected with this.

In this module we will explore various topics that underlie the deeper study of the integers, and see some initial applications. In particular, we study:

- factorisation in the integers and in other rings
- congruences and arithmetic mod n , including primitive roots
- quadratic reciprocity

- Diophantine equations, including writing integers as sums of squares

[Module web page](#)

Module aims

To introduce students to elementary number theory and provide a firm foundation for later number theory and algebra modules.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- Factorisation, divisibility, Euclidean Algorithm, Chinese Remainder Theorem.
- Congruences. Structure on $\mathbb{Z}/m\mathbb{Z}$ and its multiplicative group. Theorems of Fermat and Euler. Primitive roots.
- Quadratic reciprocity, Diophantine equations.
- Waring's problem
- Hensel's lemma
- Geometry of numbers, sum of two and four squares.

Learning outcomes

By the end of the module, students should be able to:

- Work independently with prime factorisations of integers
- Solve congruence conditions on integers
- Determine whether an integer is a quadratic residue modulo another integer
- Apply p -adic and geometry of numbers methods to solve Diophantine equations

Indicative reading list

[Reading lists can be found in Talis](#)

Subject specific skills

By the end of the module the student should be able to:

- work with prime factorisations of integers
- solve congruence conditions on integers
- determine whether an integer is a quadratic residue modulo another integer
- apply geometry of numbers methods to solve some Diophantine equations
- follow advanced courses on number theory in the third and fourth year

Transferable skills

The module will help to develop skills in understanding, assessing and constructing logical arguments (especially of a quantitative nature), and presenting these clearly in writing.

Some parts of the module will explore the difference between a theoretical solution of a problem and a solution that can be practically implemented with current computing resources, a distinction that is crucial in many real world applications of mathematical concepts.

Study

Study time

| Type | Required |
|---------------|-----------------------------|
| Lectures | 30 sessions of 1 hour (30%) |
| Seminars | 9 sessions of 1 hour (9%) |
| Private study | 61 hours (61%) |
| Total | 100 hours |

Private study description

Homework, assignments, engagement with departmental support and feedback mechanisms, exam preparation.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group B

| | Weighting | Study time | Eligible for self-certification |
|--|-----------|------------|---------------------------------|
| Centrally-timetabled examination (On-campus) | 100% | | No |

- Answerbook Pink (12 page)

Assessment group R1

| | Weighting | Study time | Eligible for self-certification |
|-------------------------------|------------------|-------------------|--|
| In-person Examination - Resit | 100% | | No |

- Answerbook Pink (12 page)

Feedback on assessment

Solutions to homework problems, support classes, solutions to exam papers.

[Past exam papers for MA357](#)

Availability

Anti-requisite modules

If you take this module, you cannot also take:

- MA257-10 Introduction to Number Theory

Courses

This module is Optional for:

- Year 4 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G100 Undergraduate Mathematics (BSc)
 - Year 3 of G100 Mathematics
 - Year 3 of G100 Mathematics
 - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
 - Year 3 of G103 Mathematics (MMath)
 - Year 3 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year