MA301-15 Waves and Metamaterials

26/27

Department
Warwick Mathematics Institute
Level
Undergraduate Level 3
Module leader
Bryn Davies
Credit value
15
Module duration
10 weeks
Assessment
100% exam
Study location
University of Warwick main campus, Coventry

Description

Introductory description

When waves propagate in heterogeneous and structured materials, an intricate combination of scattering, transmission and reflection can occur. These phenomena are the foundation of many modern wave control and manipulation devices, used in applications including telecommunications, noise control and medical imaging. These micro-structured materials are sometimes known as metamaterials. In this module, we will develop the mathematical tools needed to understand and design them.

Module aims

This module will equip students with the mathematical techniques needed to describe wave propagation and scattering in heterogeneous media. In doing so, they will learn various strategies for approximating solutions to partial differential equations, especially methods of homogenisation. We will explore how these methods are used to design wave control devices (including metamaterials such as lenses, cloaks, shields and energy harvesters).

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

The first portion of this course will be devoted to the study of waves in discrete mass-spring systems. This will be used as a toy platform to develop the key principles of wave propagation in structured materials: phase velocity, group velocity, dispersion curves and transmission and reflection coefficients.

The second portion of the course will study the one-dimensional wave equation. We will develop transfer matrix tools to study reflection and transmission by discontinuities in material parameters. Periodic materials will be considered in detail; we will use Floquet-Bloch's theorem to characterise the spectrum and study the spectral band gaps. We will use asymptotic homogenisation to characterise effective properties of the system, both at low frequencies and arbitrary non-zero frequencies.

The third section of the course will cover multi-dimensional systems. Basic properties and asymptotics of eigenvalues of the Laplacian will be studied (motivated by the result of separation of variables). Homogenisation theories for multiple dimensions will be developed. Green's functions and multiple scattering formulations will be introduced.

Finally, we will use the theory developed in this course to explore modern metamaterial applications. These will include the design of perfect (flat) lenses, gradient index lenses, graded energy harvesting devices and invisibility cloaks.

Learning outcomes

By the end of the module, students should be able to:

- Apply mathematical tools to describe wave propagation and scattering in heterogeneous media
- Use transfer matrices to describe wave propagation in one-dimensional systems
- Use asymptotic homogenisation to compute effective properties for linear waves in heterogeneous media
- Use Floquet-Bloch theory to compute spectra of periodic operators
- Understand how the eigenvalues of the Laplacian on a domain describe wave behaviour and understand their basic properties
- Understand how mathematical methods are used to design wave control devices

Indicative reading list

Reading lists can be found in Talis

Interdisciplinary

This module highlights some uses of mathematical modelling and approximation techniques in wave physics and engineering.

Subject specific skills

Students will develop an understanding of how mathematical approximation techniques can be used to model physical problems and design devices for applications. They will learn a suite of new mathematical tools, such as asymptotic homogenisation, transfer matrices, Floquet-Bloch analysis and Green’s functions. They will also learn new aspects of wave physics and how they relate to mathematical models.

Transferable skills

This module provides technical competence in characterising and approximating solutions to partial differential equations, with an emphasis on informing applications. This teaches students how to adapt a theoretical framework to real-world problems. It also develops problem solving abilities. The applications to metamaterials in the final part of the course will provide students with a case study of how mathematics is used in current cutting-edge physical applications.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	10 sessions of 1 hour (7%)
Private study	80 hours (53%)
Assessment	30 hours (20%)
Total	150 hours

Private study description

There will be weekly problem sheets for the students to complete. The solutions to these will be reviewed in the weekly support classes (tutorials).

Costs

No further costs have been identified for this module.

Assessment

You must pass all assessment components to pass the module.

Assessment group B

	Weighting	Study time	Eligible for self-certification
Assessment component			
Centrally-timetabled examination (On-campus)	100%	30 hours	No

This module will be assessed with a 3h written exam. This will include a range of questions, covering the breadth of material from the course as well as a mixture of introductory and advanced problems.

- Answerbook Gold (24 page)

Reassessment component is the same

Feedback on assessment

Students will receive their grades after the exam.

[Past exam papers for MA301]

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
  - Year 1 of G1PC Mathematics (Diploma plus MSc)
  - Year 2 of G1PC Mathematics (Diploma plus MSc)
- Year 3 of UCSA-G4G1 Undergraduate Discrete Mathematics
- UCSA-G4G3 Undergraduate Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
  - Year 3 of G4G3 Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year

This module is Core option list C for:

- Year 3 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic

and Foundations

This module is Core option list F for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- Year 3 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad
- Year 3 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 3 of GF13 Mathematics and Physics
  - Year 3 of FG31 Mathematics and Physics (MMathPhys)
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year

This module is Option list B for:

- Year 3 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)

This module is Option list C for:

- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)