MA2A4-15 Asymptotic Methods

26/27

Department
Warwick Mathematics Institute
Level
Undergraduate Level 2
Module leader
Thomasina Ball
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Asymptotics has wide ranging applications, not only to mathematical physics, but to analysis and number theory, for example. In many cases there is a need to approximate a function in a specific limit. This function may be a solution to an algebraic equation, an integral or a solution of a differential equation. Asymptotic expansions can give an excellent approximation in these limits, despite often being divergent series. Asymptotics also give approximate analytical solutions to problems with no exact analytical solution, often helping in understanding not only what happens in a particular limit, but also why.

Module aims

Introduce the subject of asymptotic methods in the contexts of algebraic functions, differential equations and integrals.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Introduction to asymptotics: Recap of big-O, little-o notation. Formal definition of an asymptotic

series. Examples, including Erf(z).

Asymptotics of algebraic equations: Regular and singular perturbations. Examples including solutions of quadratic equations.

Asymptotics of integrals: Local and non-local contributions. Exponentially dominated integrals. Watson's Lemma and Laplace’s method. Examples, including Stirling's formula.

Asymptotics of differential equations:

- Matched asymptotic expansions: Origin of small parameters (e.g. dimensionless parameters). Regular and singular perturbations. Van Dyke's matching rule. Examples of ODEs and PDEs. Logarithms and matching via intermediate variable.
- WKB Theory: WKB expansion, validity of WKB. Applications to quantum mechanics.
- Method of Multiple Scales: Resonance and secular behaviour. Multiple scale analysis. Examples include van der Pol oscillator, Mathieu equation, advection-diffusion equation.

Learning outcomes

By the end of the module, students should be able to:

- Understand the formal definition of asymptotic series and their uses.
- Be able to identify both regular and singular perturbations.
- Understand and be able to use standard techniques to construct asymptotic series for simple perturbation problems.
- Understand and be able to use more advanced techniques to construct asymptotic series in some more complicated perturbation problems, involving matched asymptotic expansions with logarithms and intermediate variables.
- Understand and be able to use WKB theory for singular differential equations.
- Understand and be able to use the method of multiple scales for differential equations with processes at different scales.

Indicative reading list

Reading lists can be found in Talis

Subject specific skills

- Ability to describe the limiting behavior of functions using "big O" and "little o" notation.
- Ability to analyze equations and identify which terms are dominant and which are negligible in a specific limit.
- Ability to derive approximate analytical solutions to problems with no exact analytical solution.
- Ability to recognise the limitations of asymptotic solutions and their region of validity.

Transferable skills

Time management. Independent Study. Logical and systematic thinking. Problem solving.

Adapting a theoretical framework to real-world problems.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Homework, assignments, engagement with departmental support and feedback mechanisms, exam preparation.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group B

	Weighting	Study time	Eligible for self-certification
Centrally-timetabled examination (on-campus)	100%		No

Assessment group R

	Weighting	Study time	Eligible for self-certification
Centrally-timetabled examination (on-campus)	100%		No

Feedback on assessment

Solutions to homework problems, support classes.

Past exam papers for MA2A4

Availability

Courses

This module is Option list A for:

- Year 2 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 2 of G100 Mathematics
    - Year 2 of G103 Mathematics (MMath)
- Year 2 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 2 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)

This module is Option list B for:

- Year 2 of UCSA-G4G1 Undergraduate Discrete Mathematics
- UCSA-G4G3 Undergraduate Discrete Mathematics
    - Year 2 of G4G1 Discrete Mathematics
    - Year 2 of G4G3 Discrete Mathematics
- Year 2 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
    - Year 2 of GF13 Mathematics and Physics
    - Year 2 of FG31 Mathematics and Physics (MMathPhys)

This module is Option list C for:

- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 2 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 2 of Y602 Mathematics,Operational Research,Stats,Economics
- Year 2 of USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics