MA299-15 Complex Methods

26/27

Department
: Warwick Mathematics Institute

Level
: Undergraduate Level 2

Module leader
: Ed Brambley

Credit value
: 15

Module duration
: 10 weeks

Assessment
: Multiple

Study location
: University of Warwick main campus, Coventry

Description

Introductory description

This is a methods module that provides an introduction to functions of a complex variable and their application to a wide range of applied maths problems. Remarkably, functions of a complex variable behave rather differently to their real counterparts, with aircraft wing design, signal processing music into its frequency content, and new ways of computing integrals and solving PDEs all depending on these properties. This course focuses on describing the theory and how it is applied, with proofs left to other modules.

Module aims

To introduce the basic ideas and theorems of complex analysis and integral transforms, and applications of these to differential and integral equations.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Complex functions: Complex numbers revision (addition, multiplication, division, complex conjugate, polar representation, etc). Entire functions, including polynomials, exponentials, and trigonometric functions. Branch points and branch cuts, with square roots as motivation. Argument and log. Fractional powers.

Complex differentiation: Definition. The Cauchy-Riemann equations. Taylor’s series and radius of convergence. Classification of isolated singularities. Meromorphic functions, and Laurent series. Analytic continuation. Conformal mappings. Mobius maps. Use of conformal maps to solve Laplace’s equation in 2D. Examples, including the Joukowsky map for airflow around aircraft wings.

Complex integration: Definition. Cauchy’s Theorem. Change of variable vs change of contour. Calculus of residues. Cauchy’s Integral Formula. Applications to evaluation of real integrals. Jordan’s lemma. Examples, including closing contours at infinity and the keyhole contour.

Integral Transforms:
a) Definition of an Integral Transform. Interpretation as a change-of-basis. Informal description of examples, including Fourier (superposition of musical notes) and Radon (CAT scans).
b) Fourier Transforms. Definition. Ambiguity in definitions (including placement of 2pi factor, sign conventions, and notation for spatial and temporal transforms). Statement of inversion formula (and non-examinable sketch of proof). Interpretation as the frequency content of a signal. Properties, including transforms of shifted functions, transforms of derivatives, transforms of products, and transformations of convolutions. Extension to more than one dimension. Applications to linear ODEs and PDEs. Examples, including the wave equation in a waveguide.
c) Laplace transforms. Properties as for Fourier Transforms. Use in solving initial-value problems for ODEs. Interpretation as half-range Fourier Transforms. Inversion. Examples.
d) Brief tour of other integral transforms, including Mellin, Z, and Radon transforms.

Learning outcomes

By the end of the module, students should be able to:

- Understand different representations of complex functions, including power series, Laurent series, their range of validity, and analytic continuation.
- Be able to classify singularities of complex functions, including poles, essential singularities, and branch points.
- Be able to apply conformal mappings to solve Laplace’s equation in 2D.
- Be able to compute contour integrals using Cauchy's integral formulas and standard contours.
- Be able to calculate relevant integral transforms and their inverses, and use their properties to solve certain differential and integral equations.

Indicative reading list

Reading lists can be found in Talis

Subject specific skills

Be comfortable working with a range of standard complex functions, including exponentials, logarithms, trigonometric functions, fractional powers, and Mobius maps.

Understand the similarities and differences between functions of a complex variable and functions of a real variable, including complex differentiability and the Cauchy-Riemann equations.

Be aware of a range of integral transforms and their interpretations.

Understand the similarities and differences between Laplace and Fourier transforms.

Transferable skills

Understand the application of complex methods in a range of applied mathematics problems.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Homework, assignments, engagement with departmental support and feedback mechansims, exam preparation.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group B

	Weighting	Study time	Eligible for self-certification
Centrally-timetabled examination (on-campus)	100%		No

Assessment group R

	Weighting	Study time	Eligible for self-certification
Centrally-timetabled examination (on-campus)	100%		No

Feedback on assessment

Solutions to homework problems, support classes.

[Past exam papers for MA299]

Availability

Courses

This module is Option list A for:

- Year 2 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 2 of G100 Mathematics
    - Year 2 of G103 Mathematics (MMath)
- Year 2 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)

This module is Option list B for:

- Year 2 of UCSA-G4G1 Undergraduate Discrete Mathematics
- UCSA-G4G3 Undergraduate Discrete Mathematics
    - Year 2 of G4G1 Discrete Mathematics
    - Year 2 of G4G3 Discrete Mathematics
- Year 2 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
    - Year 2 of GF13 Mathematics and Physics
    - Year 2 of FG31 Mathematics and Physics (MMathPhys)

This module is Option list C for:

- Year 2 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics