

MA266-10 Multilinear Algebra

26/27

Department

Warwick Mathematics Institute

Level

Undergraduate Level 2

Module leader

Gareth Tracey

Credit value

10

Module duration

10 weeks

Assessment

Multiple

Study location

University of Warwick main campus, Coventry

Description

Introductory description

It is a second Linear Algebra module, where advanced linear algebra concepts are rigorously developed for students familiar with algebraic tools.

Module aims

It will continue the study of linear algebra, which was begun in Year 1, having benefited from students finishing Abstract Algebra (Algebra-3 or Groups and Rings) in term 1.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

The module is divided in three part:

The first main topic is the Jordan canonical form and related results. Abstractly, this solves the classification problem for pairs (V, T) where V is a finite dimensional vector space over the complex numbers (or any other algebraically closed field) and T a linear self-map of V , up to the equivalence relation induced by bijective linear self-maps of V ; more concretely, we classify n by n

complex matrices A up to conjugation by invertible matrices P , i.e., the operation $A \rightarrow P^{-1}AP$.

Secondly, we treat bilinear, sesquilinear and quadratic forms on finite dimensional (real and complex) vector spaces. These structures are ubiquitous and fundamental in mathematics and many parts of the sciences. For example, the standard scalar product in \mathbb{R}^n is an example. In passing we mention that the description of amplitudes, probabilities and expectation values in quantum theory places such structures at the very heart of how nature works at the smallest levels. We will cover orthonormal basis, Gram-Schmidt process, diagonalisation, singular value decomposition, hermitian forms and normal matrices, among other things.

The third part is concerned with a thorough discussion of the very useful concept of duality (dual vector spaces, dual linear maps, dual bases etc.) and its applications, and after that tensor, exterior and symmetric algebras and their basic properties.

Learning outcomes

By the end of the module, students should be able to:

- develop full command of the theory and computation of the the Jordan canonical form of matrices and linear maps
- learn how to define and to compute functions of matrices
- develop the working knowledge of bilinear forms and quadratic forms
- master the concept of tensor and get proficient manipulating tensors

Indicative reading list

[Reading lists can be found in Talis](#)

[Specific reading list for the module](#)

Subject specific skills

This module teaches students to carry out fundamental calculations with matrices, including the theory and computation of the Jordan canonical form of matrices and linear maps; bilinear forms, diagonalizing quadratic forms, and choosing canonical bases for these. After that the module introduces the notion of tensor, treating them rigorously.

Transferable skills

The algorithmic techniques taught have widespread "real world" applications. Examples include ranking in search engines, linear programming and optimisation, signal analysis, and graphics. To also include: clear and precise thinking; the ability to follow complex reasoning; constructing logical arguments, and exposing illogical ones; and formulating problems as algorithms, thereby enhancing understanding of details and rendering them suitable for computer implementation.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (30%)
Seminars	9 sessions of 1 hour (9%)
Private study	61 hours (61%)
Total	100 hours

Private study description

Working on assignments, going over lecture notes, text books, exam revision.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D1

	Weighting	Study time	Eligible for self-certification
Assignments	15%		No
Centrally-timetabled examination (On-campus)	85%		No

- Answerbook Pink (12 page)

Assessment group R1

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

- Answerbook Pink (12 page)

Feedback on assessment

Marked homework (both assessed and formative) is returned and discussed in smaller classes. Exam feedback is given.

Availability

Courses

This module is Core for:

- Year 2 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- Year 2 of UMAA-G103 Undergraduate Mathematics (MMath)
- Year 2 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core optional for:

- Year 2 of UMAA-G100 Undergraduate Mathematics (BSc)
- Year 2 of UMAA-G103 Undergraduate Mathematics (MMath)
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-G101 Undergraduate Mathematics with Intercalated Year

This module is Optional for:

- Year 3 of USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics

This module is Option list A for:

- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- Year 2 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
 - Year 2 of GF13 Mathematics and Physics
 - Year 2 of FG31 Mathematics and Physics (MMathPhys)
- Year 2 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 2 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)

This module is Option list B for:

- Year 3 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
- Year 3 of USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics, Operational Research, Statistics, Economics (with Intercalated Year)

This module is Option list C for:

- Year 2 of USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 2 of USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics