MA254-10 Theory of ODEs

26/27

Department
Warwick Mathematics Institute
Level
Undergraduate Level 2
Module leader
Martin Lotz
Credit value
10
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Many fundamental problems in the applied sciences reduce to understanding solutions of ordinary differential equations (ODEs). Examples include the laws of Newtonian mechanics, predator-prey models in Biology, and non-linear oscillations in electrical circuits, to name only a few. These equations are often too complicated to solve exactly, so one tries to understand qualitative features of solutions.

When do solutions of ODEs exist and when are they unique? What is the long time behaviour of solutions and can they "blow-up" in finite time? These questions are answered by the Picard Theorem on existence and uniqueness of solutions of ODEs, and its consequences.

The main part of the course will focus on phase space methods. This is a beautiful geometrical approach which often enables one to understand the qualitative behaviour of solutions even when we cannot solve the equations exactly. We will develop techniques to answer important questions about the stability/attraction properties (or instabilities) of given solutions, often fixed points.

We will eventually apply these powerful methods to particular examples of practical importance, including the Lotka-Volterra model for the competition between two species, Hamiltonian systems, and the Lorenz equations, and give an informal introduction to some more advanced topics (e.g. bifurcation theory, Lyapunov exponents).

Module web page

Module aims

Extend the knowledge of first year ODEs with a mixture of applications, modelling and theory to prepare for more advanced modules later on in the course.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Introduction: The module will begin with the introduction of a few model systems to motivate questions and techniques; which will reappear throughout the module, applying the new techniques as they are acquired. Examples: Lotka-Volterra, Duffing, Lorenz, general Hamiltonian systems / nonlinear oscillator, general gradient flows.

Part I: Theory of Initial Value Problems

1. Picard Thm in R^n: concept of well-posedness, local existence and uniqueness, non-uniqueness, maximal existence interval, blowup
2. Linear theory in R^n: general solutions for constant coefficients, exponential of a matrix, variation of constants in R^n, Gronwall Lemma

Part II: Qualitative Theory of Initial Value Problems

1. Stability: linear stability, Lyapunov stability, convergence to equilibrium
2. Qualitative Theory in R^2: phase plane analysis, equilibria, local phase portraits (sketch of Hartmann-Grobman Thm), limit cycles, attractors, Bendixson-Dulac, Poincare-Bendixson)
3. Informal introduction to chaos, bifurcation, catastrophe to motivate further modules in dynamical systems (definitions and relation to applications detailed above).

Learning outcomes

By the end of the module, students should be able to:

- Determine the fundamental properties of solutions to certain classes of ODEs, such as existence and uniqueness of solutions.
- Sketch the phase portrait of 2-dimensional systems of ODEs and classify critical points and trajectories.
- Classify various types of orbits and possible behaviour of general non-linear ODEs.
- Understand the behaviour of solutions near a critical point and how to apply linearization techniques to a non-linear problem.
- Apply these methods to certain physical or biological systems.

Indicative reading list

[Reading lists can be found in Talis]

Subject specific skills

See learning outcomes

Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (30%)
Seminars	9 sessions of 1 hour (9%)
Private study	61 hours (61%)
Total	100 hours

Private study description

Homework, assignments, engagement with departmental support and feedback mechanisms, exam preparation.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group B2

	Weighting	Study time	Eligible for self-certification
Centrally-timetabled examination (On-campus)	100%		No

	Weighting	Study time	Eligible for self-certification

- Answerbook Pink (12 page)

Assessment group R2

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

- Answerbook Pink (12 page)

Feedback on assessment

Solutions to homework problems, support classes, solutions to exam papers.

[Past exam papers for MA254]

Availability

Courses

This module is Core option list A for:

- Year 2 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list B for:

- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 2 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- Year 2 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 2 of G100 Mathematics
  - Year 2 of G103 Mathematics (MMath)

- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- Year 2 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 2 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
- Year 2 of UMAA-G101 Undergraduate Mathematics with Intercalated Year

This module is Option list B for:

- Year 2 of UCSA-G4G1 Undergraduate Discrete Mathematics
- UCSA-G4G3 Undergraduate Discrete Mathematics
    - Year 2 of G4G1 Discrete Mathematics
    - Year 2 of G4G3 Discrete Mathematics
- Year 2 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
    - Year 2 of GF13 Mathematics and Physics
    - Year 2 of FG31 Mathematics and Physics (MMathPhys)

This module is Option list C for:

- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 2 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 2 of Y602 Mathematics,Operational Research,Stats,Economics
- Year 2 of USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics