

MA240-15 Modelling Nature's Nonlinearity

26/27

Department

Warwick Mathematics Institute

Level

Undergraduate Level 2

Module leader

David Wood

Credit value

15

Module duration

10 weeks

Assessment

Multiple

Study location

University of Warwick main campus, Coventry

Description

Introductory description

Many phenomena in nature can be modelled using non-linear mathematics, for example population dynamics, evolution, heartbeats, cell growth, animal locomotion, snowflakes as well as other phenomena which we cannot model with any degree of certainty, but the mathematics can help us to try and understand why certain things can happen, for example weather or apparent random properties of epidemics. The idea of this course is to introduce the techniques which mathematicians use to study, and to try to explain, the diversity and apparent unpredictability of nature. The idea is not to go through specific biological examples in any great detail, nor to rigorously prove all the techniques we use but rather to explain how the tools can be used, with some relevant examples thrown in for good measure. By the end of the course you should have enough background to be able to at least have a good guess at the techniques that should be tried when given a natural phenomena with apparent non-linear effects to model.

[Module web page](#)

Module aims

By the end of the course it is hoped that students will:

- Have a better grasp of how to develop a mathematical model, and what kind of features they would expect to see;
- Appreciate how nonlinear equations, however simple, can give rise to extremely complicated behaviour;
- Have a good background of nonlinear techniques such as Poincare Maps, extended phase space, symmetry techniques, without necessarily having to understand all of the technicalities behind them;
- Understand how models (and so 'real life' situations) can undergo sudden (catastrophic) changes after only small changes to parameters;
- Appreciate connections with the other modules they are taking (or have taken) and provided more motivation for doing them;
- Enjoy doing mathematics even more and have a good understanding of why/how Nature is nonlinear!

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Evolutionary Game Theory: Hawk-Dove and Hawk-Dove-Bully-Retaliator games used to introduce simple Game Theory techniques. Evolutionary Stable Strategies, corresponding dynamical systems, fixed points and dynamics on symplexes. Essentially how population dynamics can be explained through how individuals interact with each other.

Nonlinear Oscillations: Overview of linear oscillations (by mechanical and electrical examples), self excited oscillations (including Van Der Pol oscillator), Duffings Equation including forcing and non-forcing cases, vector fields, extended phase space, Poincare Maps, stability, Homoclinic Tangles (leading to chaos).

Epidemics: SEIR equations, discussion of contact rates and attractors for measles and chicken-pox, are such epidemics periodic or chaotic? Kermack-McKendrick model as a more analytical example.

Dynamical Systems: Discrete and continuous dynamical systems, fixed points and periodic points/orbits and stabilities. Examples include Hawk-Dove and Henon map. Invariant sets, attractors, Strange Attractors, structural stability.

Sensitive Dependence: Lyapunov Characteristic Exponents, Lyapunov Spectrum, chaotic attractors, practical method for computing LCE. Discussion on implications of chaos, and ways to 'control' chaotic systems.

Bifurcations and Catastrophes: How small changes to parameters can lead to large ('catastrophic') changes to a physical system. Euler Strut, cusp catastrophes, decision making (intelligent vs. non-intelligent), the Canonical Cusp Catastrophe, catastrophe set, singularities, the Spruce-Budworm and brief mention of the seven elementary catastrophes. Hopf bifurcation (introduced by wavy rolls in convection), linearisation and eigenvalues at a Hopf bifurcation, Hopf theorems (a Hopf bifurcation is a process which forms periodic orbits).

Coupled Oscillators: Fireflies as an example of coupled integrate and fire models, linearly coupled

relaxation oscillators, introduction to patterns observed in symmetrically coupled oscillators through the symmetries of animal gaits. Also be a discussion of how symmetry affects 'steady-state' solutions and, if time, networks of cells.

Fractals: A rather basic introduction to Fractals through the Middle Third Cantor Set, Koch Curve, Julia Sets, spaces of Fractals, chaotic repellers, iterated function systems, dimension (counting and Hausdorff dimensions).

Learning outcomes

By the end of the module, students should be able to:

- Be able to demonstrate how large changes to solutions can be caused by small changes to initial conditions or parameters.
- Show an understanding of how nonlinearity can explain many natural phenomena through mathematical modelling.
- Be able to analyse various nonlinear systems both analytically and numerically.
- Demonstrate the ability to bring together different areas of mathematics to analyse a problem both theoretical and applicable)

Indicative reading list

[Reading lists can be found in Talis](#)

Research element

20% of the assessment is the beginning of the building of a model for a problem from the natural world to demonstrate an understanding of the principal aims of the module.

Interdisciplinary

This module relies on mathematical modelling of phenomena from biology, engineering and economics.

Subject specific skills

Students will acquire a basic understanding of many mathematical concepts which can be explored further in later modules. Including, but not limited to, dynamical systems, chaos, epidemiology, game theory, symmetry, catastrophes, fractal geometry. Students will learn techniques to model phenomena in the natural world and understand how the underlying mathematics can lead to unexpected results.

Transferable skills

Mastering modelling skills of a wide range of phenomena and how to apply mathematics to areas of study outside of mathematics. Problem solving skills and appreciation of how large scale

phenomena can be affected by small changes to parameters and/or initial conditions.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	8 sessions of 1 hour (5%)
Private study	57 hours (38%)
Assessment	55 hours (37%)
Total	150 hours

Private study description

Reading lists are provided to furnish additional context to the material covered in lectures and the lecture notes. Some of the ideas will take some time to assimilate and appreciate. Significant assessment, project and revision for exam.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time	Eligible for self-certification
Modelling Project	20%	15 hours	Yes (extension)
Research and begin to model a phenomena/problem from the Natural World using techniques from the module.			
Take Home Assessment	40%	25 hours	Yes (extension)
A series of step-by-step questions that work through substantial problems.			
Centrally-timetabled examination (On-campus)	40%	15 hours	No
Exam covering aspects from all the module			

Assessment group R

	Weighting	Study time	Eligible for self-certification
Written exam	100%		No
Exam covering aspects from the module plus learning outcomes that would have been tested in the assessments.			

Feedback on assessment

Feedback will be given on individual assessments along with generic feedback to the whole cohort on common mistakes.

[Past exam papers for MA240](#)

Availability

Courses

This module is Option list A for:

- Year 2 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
 - Year 2 of G100 Mathematics
 - Year 2 of G103 Mathematics (MMath)
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics

This module is Option list B for:

- Year 2 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
 - Year 2 of GF13 Mathematics and Physics
 - Year 2 of FG31 Mathematics and Physics (MMathPhys)