ST403-15 Brownian Motion

24/25

Department
Statistics
Level
Undergraduate Level 4
Module leader
T Rosati
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Prerequisites: ST318 Probability Theory OR MA359 Measure Theory.

In 1827 the Botanist Robert Brown reported that pollen suspended in water exhibit random erratic movement. This ‘physical’ Brownian motion can be understood via the kinetic theory of heat as a result of collisions with molecules due to thermal motion. The phenomenon has later been related in Physics to the diffusion equation, which led Albert Einstein in 1905 to postulate certain properties for the motion of an idealized ‘Brownian particle’ with vanishing mass:

the path t->B(t) of the particle should be continuous
the displacements B(s+t)-B(s) should be independent of the past motion, and have a Gaussian distribution with mean 0 and variance proportional to t

[Module web page]()

Module aims

The module studies the construction and properties of Brownian motion, a fundamental tool for modelling processes which evolve randomly in time. Brownian motion is used widely in many areas of pure and applied mathematics and in the last few decades it has become essential to the study of financial maths as a model of stock prices.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

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Topics discussed in this module include:

- construction of Brownian motion/Wiener process
- fractal properties of the path, which is continuous but still a rough, non-smooth function
- description as a Gaussian process, an important class of models in machine learning
- description as a Markov process in terms of generators and semigroups
- the martingale property of Brownian motion and some aspects of stochastic calculus
- scaling properties and connection to random walk
- connection to the Dirichlet problem, harmonic functions and PDEs
- some generalizations, including e.g. geometric Brownian motion and fractional Brownian motion

Learning outcomes

By the end of the module, students should be able to:

- Apply the martingale property of Brownian Motion (BM) to derive advanced properties such as Wald's lemmas.
- Construct and explain properties of BM.
- Apply BM as a continuous time and continuous state Markov process.
- Apply the embedding of random walks in Brownian motion and use it to derive convergence results.
- Translate properties of one-dimensional BM to higher dimensions.

Indicative reading list

Peter Mörters and Yuval Peres, Brownian Motion, Cambridge University Press, 2010
René L. Schilling and Lothar Partzsch, Brownian motion: an introduction to stochastic processes, De Gruyter, 2014
Thomas M. Liggett, Continuous Time Markov Processes - An Introduction, AMS Graduate studies in Mathematics 113, 2010

View reading list on Talis Aspire

Subject specific skills

- describe its construction and explain simple properties of Brownian Motion (BM);
- understand BM as a continuous time and continuous state Markov process;
- use the martingale property of BM to derive advanced properties such as Wald’s lemmas;

- understand the embedding of random walks in Brownian motion and use it to derive convergence results;
- translate properties of one-dimensional BM to higher dimensions.

Transferable skills

- Problem solving: Use rational and logical reasoning to deduce appropriate and well-reasoned conclusions. Retain an open mind, optimistic of finding solutions, thinking laterally and creatively to look beyond the obvious. Know how to learn from failure.

- Self awareness: Reflect on learning, seeking feedback on and evaluating personal practices, strengths and opportunities for personal growth.

- Communication: Present arguments, knowledge and ideas, in a range of formats.

- Professionalism: Prepared to operate autonomously. Aware of how to be efficient and resilient. Manage priorities and time. Self-motivated, setting and achieving goals, prioritising tasks.

Study

Study time

Type	Required	Optional
Lectures	30 sessions of 1 hour (20%)	2 sessions of 1 hour
Seminars	(0%)	
Tutorials	9 sessions of 1 hour (6%)	
Private study	111 hours (74%)	
Total	150 hours	

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group D4

	Weighting	Study time	Eligible for self-certification
Assignments worth 15% Coursework	15%		No
In-person Examination	85%		No

The examination paper will contain five questions, of which the mark from the FIRST question and the best marks of THREE of the remaining four questions will be used to calculate your grade.

- Answerbook Pink (12 page)

Assessment group R3

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

The examination paper will contain five questions, of which the mark from the FIRST question and the best marks of THREE of the remaining four questions will be used to calculate your grade.

- Answerbook Pink (12 page)

Feedback on assessment

Marked coursework and exam feedback

Past exam papers for ST403

Availability

Anti-requisite modules

If you take this module, you cannot also take:

- MA4F7-15 Brownian Motion

Courses

This module is Optional for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 1 of TIBS-N3G1 Postgraduate Taught Financial Mathematics
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
    - Year 1 of G1PC Mathematics (Diploma plus MSc)
    - Year 2 of G1PC Mathematics (Diploma plus MSc)
- Year 4 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics

This module is Option list A for:

- Year 4 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- Year 5 of USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Option list B for:

- Year 4 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 5 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)

This module is Option list C for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
    - Year 4 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad

This module is Option list D for:

- Year 4 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- Year 5 of USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated

This module is Option list E for:

- Year 4 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- Year 5 of USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated