

PX286-10 Methods of Mathematical Physics

24/25

Department

Physics

Level

Undergraduate Level 2

Module leader

Rudo Roemer

Credit value

10

Module duration

10 weeks

Assessment

Multiple

Study location

University of Warwick main campus, Coventry

Description

Introductory description

The module covers the theory of Fourier transforms and the Dirac delta function. The module also introduces Lagrange multipliers, co-ordinate transformations and cartesian tensors illustrating them with examples of their use in physics.

Fourier transforms are used to represent functions on the whole real line using linear combinations of sines and cosines. A Fourier transform will turn a linear differential equation with constant coefficients into a nice algebraic equation which is in general easier to solve. The module explains why diffraction patterns in the far-field limit are the Fourier transforms of the "diffracting" object. The case of a repeated pattern of motifs illustrates beautifully one of the most important theorems in the business - the convolution theorem. The diffraction pattern is the product of the Fourier transform of repeated delta functions and the Fourier transform for a single copy of the motif.

[Module web page](#)

Module aims

To teach mathematical techniques needed by second, third and fourth year physics modules

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Fourier Series:

Representation for function $f(x)$ defined $-L$ to L ; mention of convergence issues; real and complex forms; differentiation, integration; periodic extensions

2. Fourier Transforms:

Fourier series when L tends to infinity. Definition of Fourier transform and standard examples - Gaussian, exponential and Lorentzian.

Domains of application: (Time t | frequency w), (Space x | wave vector k).

Delta function and properties, Fourier's Theorem.

Convolutions, example of instrument resolution, convolution theorem

3. Interference and diffraction phenomena:

Interference and diffraction, the Huygens-Fresnel principle. Criteria for Fraunhofer and Fresnel

diffraction. Fraunhofer diffraction for parallel light. Fourier relationship between an object and its diffraction pattern. Convolution theorem demonstrated by diffraction patterns. Fraunhofer diffraction for single, double and multiple slits. Fraunhofer diffraction at a circular aperture; the

Airy disc. Image resolution, the Rayleigh criterion and other resolution limits. Fresnel diffraction,

shadow edges and diffraction at a straight edge

4. Lagrange Multipliers:

Variation of $f(x,y)$ subject to $g(x,y) = \text{constant}$ implies $\text{grad } f$ parallel to $\text{grad } g$. Lagrange multipliers. Example of quadratic form

5. Vectors and Coordinate Transformations:

Summation convention, Kronecker delta, permutation symbol and use for representing vector

products. Revision of cartesian coordinate transformations. Diagonalizing quadratic forms

6. Tensors:

Physical examples of tensors: mass, current, conductivity, electric field

7. Stokes' Theorem (Worksheet):

Line integrals, circulation; curl in Cartesians; statement and proof of Stokes' theorem for triangulations; dependence on region of integration and vector field; gradient, irrotational, solenoidal and incompressible vector fields; applications drawn from electromagnetism, fluid dynamics, condensed matter physics, differential geometry

Learning outcomes

By the end of the module, students should be able to:

- Represent functions in terms of Fourier series and Fourier transforms
- Demonstrate understanding of diffraction and interference phenomena
- Minimise/maximise functions subject to constraints using Lagrange multipliers
- Express vectors in different coordinate systems, recognise some physical examples of

tensors

- Derive, and apply in physical contexts, Stokes's theorem

Indicative reading list

KF Riley, MP Hobson and SJ Bence, *Mathematical Methods for Physics and Engineering: a Comprehensive Guide*, Wadsworth, H D Young and R A Freedman, *University Physics 11th Edition*, Pearson.

[View reading list on Talis Aspire](#)

Subject specific skills

Mathematical methods including: Fourier transforms and their application to describe diffraction, Lagrange multipliers.

Skills in modelling, reasoning, thinking

Transferable skills

Analytical, communication, problem-solving, self-study

Study

Study time

| Type | Required |
|----------------|-----------------------------|
| Lectures | 20 sessions of 1 hour (20%) |
| Other activity | 10 hours (10%) |
| Private study | 70 hours (70%) |
| Total | 100 hours |

Private study description

Working through lecture notes, solving problems, wider reading, discussing with others taking the module, revising for exam, practising on past exam papers

Other activity description

10 example classes

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

| | Weighting | Study time |
|---------------------------------|-----------|------------|
| Assessed Coursework | 20% | |
| Methods of Mathematical Physics | 80% | |
| Answer 2 questions | | |

- Answerbook Green (8 page)
- Students may use a calculator

Assessment group R

| | Weighting | Study time |
|-------------------------------|-----------|------------|
| In-person Examination - Resit | 100% | |
| Answer 2 questions | | |

- Answerbook Green (8 page)
- Students may use a calculator

Feedback on assessment

Personal tutors, group feedback

[Past exam papers for PX286](#)

Availability

Courses

This module is Core for:

- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
 - Year 2 of GF13 Mathematics and Physics
 - Year 2 of GF13 Mathematics and Physics

- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
 - Year 2 of FG31 Mathematics and Physics (MMathPhys)
 - Year 2 of FG31 Mathematics and Physics (MMathPhys)

This module is Option list B for:

- Year 2 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G100 Undergraduate Mathematics (BSc)
 - Year 2 of G100 Mathematics
 - Year 2 of G100 Mathematics
 - Year 2 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
 - Year 2 of G100 Mathematics
 - Year 2 of G103 Mathematics (MMath)
 - Year 2 of G103 Mathematics (MMath)
- Year 2 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- Year 2 of UMAA-G101 Undergraduate Mathematics with Intercalated Year