MA9N7-15 Topics in Interacting Particle Systems

24/25

Department
: Warwick Mathematics Institute

Level
: Research Postgraduate Level

Module leader
: Oleg Zaboronski

Credit value
: 15

Module duration
: 10 weeks

Assessment
: 100% exam

Study location
: University of Warwick main campus, Coventry

Description

Introductory description

The theory of interacting particle systems is often understood as a synonym for the 'theory of everything' covering any classical particle system with many degrees of freedom. Examples range from the evolution of galaxies in the Universe to kinetic theory of gases and spin chains in physics; from epidemiology to synchronisation of schools of fish to the dynamics of insect and bacterial colonies in biology; agent-based models in economics, social sciences and finances; Boltzman-Ising annealers in quantum computing; reaction-diffusion systems of molecules in chemistry.

This course will be dedicated to the rigorous study of interacting particle systems (IPS) as a branch of probability theory. For us, an IPS is a continuous time Markov process on the configuration space O=S^G, where S is a finite set and G is a graph. Typically, G=Z^d, a hypercubic lattice. Surprisingly many examples from the above list can be modelled using the language of Markov processes on O.

We will study IPS's starting with foundational questions such as their rigorous definitions and existence proofs, progressing to general methods of study including coupling, duality, convergence to steady state. A certain emphasis will be placed on the study of strong fluctuation effects for interacting particle systems when various mean field approximations fail completely. These effects dominate the large-scale behaviour of low-dimensional particle systems. Typically, these are IPS's on Z^d, where d=1,2.

Module aims

-Learn to model a physical IPS as a continuous time Markov process on a possibly uncountable configuration space
-Prove existence theorems for IPS's
-Learn duality and coupling methods of studying IPS's with application to convergence to a steady state.
-Describe a one-dimensional law of an IPS as a point process. Learn about special point processes - Poisson, Pfaffian and determinantal
-Use duality and the theory of point processes to study fluctuation effects for certain IPS in one dimension

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Possible topics comprise:
-The fundamentals of continuous time Markov chains on non-countable configuration spaces (Well-posedness theorems, the generator formalism, coupling and duality, analysis of invariant measures and convergence to IM's. Working examples: voter models, contact processes, exclusion models.)
-Introduction to point processes (definition of simple point processes, correlation functions, Janossi densities, gap probabilities, determinantal and Pfaffian point processes,
Fredholm determinants and Pfaffians, Szego-type theorems for asymptotic computation of functional determinants. Running examples: Poisson PP, one-dependent point processes, PP's appearing in RMT, random polynomials)

- Reaction-diffusion models in one dimension (annihilating coalescing random walks, branching-coalescing random walks, full set of duality functions, Pfaffian point processes. Large deviations results for the gap size distributions. Continuous limits: the point sets for the Brownian Web and the Brownian Net.)

Learning outcomes

By the end of the module, students should be able to:

- Apply the foundational material covered in the course to the rigorous study of interaction particle systems
- Apply techniques that have been developed in the course to extract detailed information about the large-scale statistics of IPS
- Provide students with enough theoretical material and examples to enable confident modelling and analysis of processes appearing in physics, biology, economics, quantum computing as IPS's

Subject specific skills

1. Apply the foundational material covered in the course to the rigorous study of interaction particle systems
2. Apply techniques that have been developed in the course to extract detailed information about the large-scale statistics of IPS
3. Provide students with enough theoretical material and examples to enable confident modelling and analysis of processes appearing in physics, biology, economics, quantum computing as IPS's

Transferable skills

- sourcing research material
- prioritising and summarising relevant information
- absorbing and organizing information
- presentation skills (both oral and written)

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	10 sessions of 1 hour (7%)
Private study	80 hours (53%)
Assessment	30 hours (20%)
Total	150 hours

Private study description

Review lectured material.
Work on suplementary reading material.
Source, organise and prioritise material for additional reading.

Costs

No further costs have been identified for this module.

Assessment

You must pass all assessment components to pass the module.

Assessment group B

	Weighting	Study time
Oral Exam	100%	30 hours

An oral exam involving a presentation by the student, followed by questions from the panel (2 members of the department)

Feedback on assessment

Students will receive feedback from the course instructor after the oral exam, to cover also areas like presentation skills and use of technologies (or blackboard)

Past exam papers for MA9N7

Availability

There is currently no information about the courses for which this module is core or optional.