MA4N3-15 Hyperbolic Dynamics

24/25

Department
Warwick Mathematics Institute
Level
Undergraduate Level 4
Module leader
Stephen Cantrell
Credit value
15
Module duration
10 weeks
Assessment
100% exam
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Hyperbolic Dynamics is a very active area of the field of Dynamical Systems. This course will be an introduction to the subject focusing initially accessible examples which still illustrate the important features of the theory: expanding circle maps, hyperbolic toral automorphisms, Smale horseshoe and Smale solenoid. We will develop key theoretical aspects of the theory of hyperbolic dynamical systems, More advanced topics will be selected from equidistribution theory for periodic points, hyperbolic flows, and examples on homogeneous spaces.

Module web page

Module aims

This module will have three main strands. First, to understand how hyerbolicity gives rise to complicated dynamical behaviour via the study of accessible examples. Second, to understand the general theory of hyperbolic systems and their properties, such as structural stability, symbolic dynamics and stable manifold theory, and introduce and apply the notion of topological entropy in this setting. Third, to understand more advanced topics and examples, such as equidistribution theory for periodic points, hyperbolic flows, and examples on homogeneous spaces.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

We will cover some of the following topics:

1. Expanding maps: expanding circle maps, degree symbolic dynamics, classification, statement of Groom's classification theorem, piecewise expanding Markov interval maps, subshifts of finite type.
2. Hyperbolic toral automorphisms, Anosov systems, Axion A systems, Smale horseshoe, Smale solenoid, symbolic dynamics, explicit construction of Markov partitions for simple examples.
3. Stable manifold theory, shadowing, structural stability.
4. Topological entropy, Bowen-Dinaburg definition, invariance under topological conjugacy, calculation for hyperbolic systems.
5. Equidistribution of periodic points for hyperbolic toral automorphisms, zeta functions.
6. Hyperbolic flows, geodesic flows, suspension flows.
7. Flows on matrix groups and homogeneous spaces, connections to number theory.

Learning outcomes

By the end of the module, students should be able to:

- Use a variety of analytic and geometric techniques to analyse hyperbolic dynamical systems.
- Understand the role of structural stability in dynamical systems.
- Understand the role of symbolic dynamics in the theory of hyperbolic dynamical systems.
- Understand how topological entropy can be calculated.
- Understand how to study the distribution of orbits of hyperbolic dynamical systems.

Subject specific skills

Students will have an in depth knowledge of the theory of hyperbolic dynamical systems and of specific examples. These systems all exhibit chaotic behaviour and students will be able to understand and analyse their properties., and will have become equipped with a variety of analytic and geometric tools to do this. They will have the background to begin postgraduate study in the area, and to apply their knowledge to others areas of dynamics.

Transferable skills

Students will be equipped to understand a range of dynamical systems exhibiting complicated behaviour, and to understand that such behaviour may be stable under perturbation. They will have the background to study various areas of applications where dynamical systems appear, such as control systems, engineering and meteorology. More generally, they will have had the opportunity to develop their analytic skills through the study of systems which are complicated but which are amenable to rigorous study.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	9 sessions of 1 hour (6%)
Tutorials	9 sessions of 1 hour (6%)
Private study	59 hours (39%)
Assessment	43 hours (29%)
Total	150 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group B

	Weighting	Study time	Eligible for self-certification
Assessment component			
In-person Examination 3 hour exam, no books allowed	100%	43 hours	No

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- Answerbook Pink (12 page)

Reassessment component is the same

Feedback on assessment

Marked assignments and exam feedback.

Past exam papers for MA4N3

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
    - Year 1 of G1PC Mathematics (Diploma plus MSc)
    - Year 2 of G1PC Mathematics (Diploma plus MSc)

This module is Option list B for:

- Year 4 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 5 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)

This module is Option list C for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
    - Year 4 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad
- Year 4 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe