MA3J8-15 Approximation Theory and Applications

24/25

Department
Warwick Mathematics Institute
Level
Undergraduate Level 3
Module leader
Clarice Poon
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

This module will provide students with a foundation in approximation theory, driven by its applications in scientific computing and data science.

Module web page

Module aims

In approximation theory a function that is difficult or impossible to evaluate directly, e.g., an unknown constitutive law or the solution of a PDE, is to be approximated as efficiently as possible from a more elementary class of functions, the approximation space. The module will explore different choices of approximation spaces and how they can be effective in different applications chosen from typical scientific computing and data science, including e.g. global polynomials, trigonometric polynomials, splines, radial basis functions, ridge functions (neural networks) as well as methods to construct the approximations, e.g., interpolation, least-squares.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Part 1: univariate approximation

- spline approximation of smooth functions in 1D
- polynomial and trigonometric approximation of analytic functions in 1D
- linear best approximation

Part 2: Multi-variate approximation: details will depend on the progress through Part 1 and available time, but the idea of Part 2 is to cover a few selected examples of high-dimensional approximation theory, for example a sub-set of the following:
wavelets and nonlinear approximation,
applications to image processing, such as image compression,
compressed sensing and best n-term approximation,
ridge functions and neural networks.

Throughout the lecture each topic will cover (1) approximation rates, (2) algorithms, and (3) examples, typically implemented in Python. Any programming aspects of the module will not be examinable.

Learning outcomes

By the end of the module, students should be able to:

- demonstrate understanding of key concepts, theorems and calculations of univariate approximation theory.
- demonstrate understanding of a selection of the basic concepts, theorems and calculations of multivariate approximation theory.
- demonstrate understanding of basic algorithms and examples used in approximation theory.

Indicative reading list

Reading lists can be found in Talis

Subject specific skills

Formalisation of function approximation, connection to applications in modelling, numerical analysis and machine learning. Specific topics:

- Abstract approximation in Hilbert and Banach spaces
- Connect approximation rates and function regularity, in particular analyticity
- Approximation by trigonometric and algebraic polynomials
- Multi-variate approximation
- Approximation via Least Squares Fitting
- High-dimensional problems
- Practical implementation of algorithms
- Introduction to wavelets and applications to image processing.

Transferable skills

Analytical thinking and problem solving: Numerical models and methods of the kind that will be studied are crucial in many areas of science, engineering, and machine learning. The course trains students to connect rigorous mathematical ideas with concrete numerical algorithms, programming and scientific computing. E.g. it will train students to better select suitable computational tools for approximation and inverse problems with are ubiquitous across many disciplines.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Independent study, non-assessed example sheets and revision for exam.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group B2

	Weighting	Study time	Eligible for self-certification
Centrally-timetabled examination (On-campus) A 3-hour written exam.	100%		No

	Weighting	Study time	Eligible for self-certification

- Answerbook Pink (12 page)

Assessment group R1

	Weighting	Study time	Eligible for self-certification
Exam	100%		No

- Answerbook Gold (24 page)

Feedback on assessment

Exam feedback

Past exam papers for MA3J8

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
  - Year 1 of G1PC Mathematics (Diploma plus MSc)
  - Year 2 of G1PC Mathematics (Diploma plus MSc)
- Year 3 of UCSA-G4G1 Undergraduate Discrete Mathematics
- UCSA-G4G3 Undergraduate Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
  - Year 3 of G4G3 Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- Year 3 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics

This module is Core option list A for:

- Year 4 of UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year

This module is Core option list C for:

- Year 3 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list F for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
    - Year 4 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- Year 3 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- Year 3 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G100 Mathematics
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of USTA-G140 Undergraduate Mathematics and Statistics (MMathStat)
- Year 5 of USTA-G141 Undergraduate Mathematics and Statistics (MMathStat) (with Intercalated Year)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- Year 3 of USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 3 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
- Year 4 of USTA-G141 Undergraduate Mathematics and Statistics (MMathStat) (with Intercalated Year)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

This module is Option list C for:

- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 3 of USTA-G140 Undergraduate Mathematics and Statistics (MMathStat)