MA3H3-15 Set Theory

24/25

Department
Warwick Mathematics Institute
Level
Undergraduate Level 3
Module leader
Andras Mathe
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

See learning outcomes.

Module web page

Module aims

Set theoretical concepts and formulations are pervasive in modern mathematics. For this reason it is often said that set theory provides a foundation for mathematics. Here 'foundation' can have multiple meanings. On a practical level, set theoretical language is a highly useful tool for the definition and construction of mathematical objects. On a more theoretical level, the very notion of a foundation has definite philosophical overtones, in connection with the reducibility of knowledge to agreed first principles.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

The module will commence with a brief review of naive set theory. Unrestricted set formation leads

to various paradoxes (Russell, Cantor, Burali-Forti), thereby motivating axiomatic set theory. The Zermelo-Fraenkel system will be introduced, with attention to the precise formulation of axioms and axiom schemata, the role played by proper classes, and the cumulative hierarchy picture of the set-theoretical universe. Transfinite induction and recursion, cardinal and ordinal numbers, and the real number system will all be developed within this framework. The Axiom of Choice, and various equivalents and consequences, will be discussed, and various other principles known to be ZF-independent, such as the Continuum Hypothesis and the existence of Inaccessible Cardinals, will also be touched on.

Learning outcomes

By the end of the module, students should be able to:

- Formally state the axioms of Zermelo-Fraenkel set theory.
- Rigourously compare sizes and orderings of sets by means of explicit constructions of injections and bijections, and give interpretations in the terminology of cardinal and ordinal arithmetic.
- Outline the construction of the real number system, though various stages, ultimately from first principles.
- Give examples of mathematical statements which are equivalent to the Axiom of Choice, notice the use of this principle in mathematical arguments, and avoid unnecessary use of it.
- Appreciate the strengths, and also some of the shortcomings, of Zermelo-Fraenkel set theory as a foundation for mathematics.

Indicative reading list

Set Theory, Jech
This is a comprehensive advanced text which goes well beyond
the above syllabus.
Notes on set theory, Y. Moschovakis
Elements of set theory, H. Enderton
Introduction to set theory, Hrbacek and Jech

Subject specific skills

Appreciate the strengths, and also some of the shortcomings, of Zermelo-Fraenkel set theory as a foundation for mathematics.

Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	10 sessions of 1 hour (7%)
Private study	110 hours (73%)
Total	150 hours

Private study description

Office hours, private study, preparation for lectures and exams, assignment sheets - 110 hours

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group B1

	Weighting	Study time	Eligible for self-certification
In-person Examination	100%		No

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

- Answerbook Gold (24 page)

Feedback on assessment

Exam Feedback

Past exam papers for MA3H3

Availability

Courses

This module is Core for:

- Year 3 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 4 of UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Optional for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
    - Year 1 of G1PC Mathematics (Diploma plus MSc)
    - Year 2 of G1PC Mathematics (Diploma plus MSc)
- Year 3 of UCSA-G4G1 Undergraduate Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics

This module is Core option list B for:

- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
    - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
    - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
    - Year 3 of G105 Mathematics (MMath) with Intercalated Year

    - Year 4 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- Year 3 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G100 Mathematics
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G107 Undergraduate Mathematics (MMath) with Study Abroad
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
    - Year 3 of G106 Mathematics (MMath) with Study in Europe
    - Year 4 of G106 Mathematics (MMath) with Study in Europe
- Year 3 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- Year 3 of UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- Year 3 of USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics,Operational Research,Statistics,Economics (with Intercalated Year)

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 3 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

This module is Option list C for:

- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)

This module is Option list E for:

- Year 4 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
    - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)