## MA398-15 Matrix Analysis \& Algorithms

## 24/25

## Department

Warwick Mathematics Institute

## Level

Undergraduate Level 3
Module leader
Randa Herzallah
Credit value
15
Module duration
10 weeks

## Assessment

Multiple

## Study location

University of Warwick main campus, Coventry

## Description

## Introductory description

Many large scale problems arising in data analysis and scientific computing require efficient solutions to systems of
linear equations, least-squares problems, and eigenvalue problems. The module is based around understanding the
mathematical principles underlying the design and the analysis of effective methods and algorithms to solve these
types of challenges. It involves an interplay between mathematical analysis and computing, including programming aspects, validation and result analysis.

## Module web page

## Module aims

Understanding how to construct algorithms for solving problems central in numerical linear algebra and how to
analyse them with respect to accuracy and computational cost. Selecting, evaluating and
characterising findings when
applying both analytical and numerical methods to practical problems of interest.

## Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Many large scale problems arising in data analysis and scientific computing require efficient solutions to systems of
linear equations, least-squares problems, and eigenvalue problems. The module is based around understanding the
mathematical principles underlying the design and the analysis of effective methods and algorithms to solve these
types of challenges. Key concepts include matrix decompositions, factorisations, direct and iterative system solvers.
They are supported by fundamental aspects related to floating point arithmetic, computational cost analysis, stability
and convergence, with examples such as image compression and operating in data-rich environments. Incursions into
ideas from high performance computing will also feed into the material in order to inform about modern software
engineering practices in academia and beyond. This module is suitable for students in joint degrees with Mathematics, as well as associated departments such as (but not limited to) Computer Science, Physics and Statistics.

## Learning outcomes

By the end of the module, students should be able to:

- explain concepts and ideas related to matrix factorisations as the theoretical basis for algorithms,
- assess algorithms with respect to computational cost, conditioning of problems and stability of algorithms,
- demonstrate theoretical concepts using numerical software tools,
- compare different (competing) numerical methodologies and critically evaluate bottlenecks in algorithmic design for real-world applications.


## Indicative reading list

AM Stuart and J Voss, Matrix Analysis and Algorithms, script.
G Golub and C van Loan, Matrix Computations, 3. ed., Johns Hopkins Univ. Press, London 1996.
NJ Higham, Accuracy and Stability of Numerical Algorithms, SIAM 1996.
RA Horn and CR Johnson, Matrix Analysis, Cambridge University Press 1985.
D Kincaid and W Cheney, Numerical Analysis, 3. ed., AMS 2002.
LN Trefethen and D Bau, Numerical Linear Algebra, SIAM 1997.

The module sits naturally between mathematics, computer science and elements from related disciplines such as physics, biology or image processing.

## Subject specific skills

Discrete mathematical analysis, algorithmic construction, cost and error analysis, concepts in numerical methods for a range of applied problems in mathematical sciences.

## Transferable skills

Informed problem solving, software development, creative solution analysis, project management, critical thinking, application-oriented solution strategy design, programming language expertise.

## Study

## Study time

| Type | Required |
| :--- | :--- |
| Lectures | 30 sessions of 1 hour (20\%) |
| Tutorials | 9 sessions of 1 hour (6\%) |
| Private study | 111 hours $(74 \%)$ |
| Total | 150 hours |

## Private study description

Each assignment should take approximately 8 h to complete, in line with its weight in the overall module assessment.
Several hours of preparation (software installation, any debugging e.g. using support classes) will also likely be
necessary. Lecture preparation (20h), engaging with online material (20h), participation in optional seminar
discussions (10h) and work on formative elements (15-20h, be they pen and paper based or programming) is
anticipated to fill up the rest of the independent learning time, though many of the above offer checkpoints in terms of lectures, support classes or TA-run support classes.

## Costs

No further costs have been identified for this module.

## Assessment

You must pass all assessment components to pass the module.
Students can register for this module without taking any assessment.

## Assessment group D2

|  | Weighting Study time |
| :--- | :--- |
| Assignment | $15 \%$ |

Three assignments (posted in weeks 2,5 and 8 ), with feedback provided within 2 weeks from submission, so as to enable feedforward into the following assignment and ultimately study. These
will consist of $70 \%$ theoretical work (theorem proving and application) and $30 \%$ implementation, data visualisation and interpretation, with a final component attributed to brief discussions about the wider implications of the material.

## In-person Examination 85\%

Pen and paper work only, with a combination of theoretical content (theorem proving, comparisons etc.), application to particular examples (or finding counterexamples), filling in or improving provided algorithmic scaffolding, as well as critical assessment of provided results. The precise format depends on the nature of the assessment (online versus in person).

- Answerbook Gold (24 page)


## Assessment group R

Weighting Study time
In-person Examination - Resit $100 \%$
Pen and paper work only, with a combination of theoretical content (theorem proving,
comparisons etc.), application to particular examples (or finding counterexamples), filling in or
improving provided algorithmic scaffolding, as well as critical assessment of provided results. The
precise format depends on the nature of the assessment (online versus in person).

- Answerbook Gold (24 page)


## Feedback on assessment

Formative assessments (exercise sheets, $3 x$ during the term), as well as lecture materials are discussed in Q\&A sessions with the module leader, as well as in support classes with the TA.

Timely feedback will be provided after each summative assessment (assignment sheets, $3 x$ during the term) such that
it can be incorporated in the developing study process. This will include comments on both mathematical and good
programming practice, such that the outcomes can propagate beyond the level of the module itself.

## Past exam papers for MA398

## Availability

## Pre-requisites

MA106 Linear Algebra and (to a lesser extent) MA259 Multivariable Calculus are sufficient in terms of core 1st and
2nd year modules within Mathematics. Helpful but not mandatory is some knowledge of numerical concepts such as
accuracy, iteration processes, and stability as provided in MA228 Numerical Analysis or MA261
Differential Equations:
Modelling and Numerics.

## Courses

This module is Optional for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UCSA-G4G1 Undergraduate Discrete Mathematics

Year 3 of G4G1 Discrete Mathematics
Year 3 of G4G1 Discrete Mathematics

- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics

Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy

Year 3 of GV17 Mathematics and Philosophy
Year 3 of GV17 Mathematics and Philosophy
Year 3 of GV17 Mathematics and Philosophy

- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic

This module is Core option list D for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year Year 4 of GV18 Mathematics and Philosophy with Intercalated Year Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list $A$ for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)

Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)

- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- Year 1 of G1PC Mathematics (Diploma plus MSc)
- Year 2 of G1PC Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- Year 3 of G105 Mathematics (MMath) with Intercalated Year

Year 4 of G105 Mathematics (MMath) with Intercalated Year
Year 5 of G105 Mathematics (MMath) with Intercalated Year

- UMAA-G100 Undergraduate Mathematics (BSc)

Year 3 of G100 Mathematics
Year 3 of G100 Mathematics
Year 3 of G100 Mathematics

- UMAA-G103 Undergraduate Mathematics (MMath)

Year 3 of G100 Mathematics
Year 3 of G103 Mathematics (MMath)
Year 3 of G103 Mathematics (MMath)
Year 4 of G103 Mathematics (MMath)
Year 4 of G103 Mathematics (MMath)

- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe

Year 3 of G106 Mathematics (MMath) with Study in Europe Year 4 of G106 Mathematics (MMath) with Study in Europe

- Year 3 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- Year 3 of GF13 Mathematics and Physics Year 3 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)

Year 3 of FG31 Mathematics and Physics (MMathPhys)
Year 3 of FG31 Mathematics and Physics (MMathPhys)

- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)

Year 3 of GG14 Mathematics and Statistics
Year 3 of GG14 Mathematics and Statistics

- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics

Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

This module is Option list $E$ for:

- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics

Year 3 of G30D Master of Maths, Op.Res, Stats \& Economics (Statistics with Mathematics Stream)
Year 4 of G30D Master of Maths, Op.Res, Stats \& Economics (Statistics with Mathematics Stream)

- USTA-G301 Undergraduate Master of Mathematics, Operational Research,Statistics and Economics (with Intercalated

Year 3 of G30H Master of Maths, Op.Res, Stats \& Economics (Statistics with Mathematics Stream)
Year 4 of G30H Master of Maths, Op.Res, Stats \& Economics (Statistics with Mathematics Stream)
Year 5 of G30H Master of Maths, Op.Res, Stats \& Economics (Statistics with Mathematics Stream)

