MA357-10 Introduction to Number Theory

24/25

Department
Warwick Mathematics Institute
Level
Undergraduate Level 3
Module leader
Sam Chow
Credit value
10
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Number theory is an ancient and beautiful subject that investigates the properties of the integers, and other questions motivated by them. For example, one studies questions like:

- do equations have integer solutions? If so, do they have infinitely many integer solutions? How are those solutions distributed, and can we parametrise them? Pythagorean triples, and Fermat's Last Theorem, fall into this general area of Diophantine equations.
- how do special subsets of the integers behave? The primes are probably the best example of a special subset, and one wants to know: are there infinitely many? How are they distributed? Are there interesting patterns amongst them? The most famous unsolved problem in mathematics, the Riemann Hypothesis, is connected with this.

In this module we will explore various topics that underlie the deeper study of the integers, and see some initial applications. In particular, we study:

- factorisation in the integers and in other rings
- congruences and arithmetic mod n, including primitive roots
- quadratic reciprocity

- Diophantine equations, including writing integers as sums of squares

Module web page

Module aims

To introduce students to elementary number theory and provide a firm foundation for later number theory and algebra modules.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- Factorisation, divisibility, Euclidean Algorithm, Chinese Remainder Theorem.
- Congruences. Structure on Z/mZ and its multiplicative group. Theorems of Fermat and Euler. Primitive roots.
- Quadratic reciprocity, Diophantine equations.
- Waring's problem
- Hensel's lemma
- Geometry of numbers, sum of two and four squares.

Learning outcomes

By the end of the module, students should be able to:

- Work independently with prime factorisations of integers
- Solve congruence conditions on integers
- Determine whether an integer is a quadratic residue modulo another integer
- Apply p-adic and geometry of numbers methods to solve Diophantine equations

Indicative reading list

H. Davenport, The Higher Arithmetic, Cambridge University Press.

G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers, Oxford University Press, 1979.

K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, Springer-Verlag, 1990.

Subject specific skills

By the end of the module the student should be able to:

- work with prime factorisations of integers
- solve congruence conditions on integers
- determine whether an integer is a quadratic residue modulo another integer

- apply geometry of numbers methods to solve some Diophantine equations
- follow advanced courses on number theory in the third and fourth year

Transferable skills

The module will help to develop skills in understanding, assessing and constructing logical arguments (especially of a quantitative nature), and presenting these clearly in writing.

Some parts of the module will explore the difference between a theoretical solution of a problem and a solution that can be practically implemented with current computing resources, a distinction that is crucial in many real world applications of mathematical concepts.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (30%)
Seminars	9 sessions of 1 hour (9%)
Private study	61 hours (61%)
Total	100 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time
Assignment	15%	
Examination	85%	

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Pink (12 page)

Feedback on assessment

Support Classes
Marked homework will be returned to students.
Exam feedback.

Past exam papers for MA357

Availability

Anti-requisite modules

If you take this module, you cannot also take:

- MA257-10 Introduction to Number Theory

Courses

This module is Core option list A for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year

This module is Core option list C for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list F for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 4 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G100 Undergraduate Mathematics (BSc)
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G100 Mathematics
    - Year 3 of G103 Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
    - Year 3 of GF13 Mathematics and Physics
    - Year 3 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
    - Year 3 of GF13 Mathematics and Physics
    - Year 3 of FG31 Mathematics and Physics (MMathPhys)
    - Year 3 of FG31 Mathematics and Physics (MMathPhys)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year