MA269-10 Asymptotics and Integral Transforms

24/25

Department
: Warwick Mathematics Institute

Level
: Undergraduate Level 2

Module leader
: Thomasina Ball

Credit value
: 10

Module duration
: 10 weeks

Assessment
: Multiple

Study location
: University of Warwick main campus, Coventry

Description

Introductory description

A two-part course covering an introduction to asymptotics, and an introduction to integral transforms, focusing on their properties and their applications, with proofs to come in later courses (although these could be hinted at by the lecturer). The course covers standard techniques that are of widespread use throughout applied mathematics, physics, and engineering.

Module aims

The course is in two parts. The first half covers an introduction to asymptotics. The second half covers an introduction to integral transforms.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Asymptotics:

a) Formal definition of an asymptotic series. Examples, including Erf(z). Discussion of the origins of small parameters (e.g. dimensionless parameters, stability analysis)
b) Asymptotics of algebraic equations, including singular perturbations. Examples including solutions of quadratic equations.
c) Asymptotics of integrals. Local and non-local contributions. Watson's Lemma. Steepest Descents. Examples, including Stirling's formula. Application of Steepest Descent contours to the (exact) computation of oscillatory integrals.
d) Asymptotics of differential equations. Regular and singular perturbations. Matched Asymptotic Expansions in simple cases only. Van Dyke's matching rule. Examples

2. Integral Transforms
a) Definition of an Integral Transform. Interpretation as a change-of-basis. Informal description of examples, including Fourier (superposition of musical notes) and Radon (CAT scans).
b) Fourier Transforms. Definition. Ambiguity in definitions (including placement of 2pi factor, sign conventions, and notation for spatial and temporal transforms). Statement of inversion formula (and non-examinable sketch of proof). Interpretation as the frequency content of a signal. Properties, including transforms of shifted functions, transforms of derivatives, transforms of products, and transformations of convolutions. Extension to more than one dimension. Applications to linear ODEs and PDEs. Examples, including the wave equation in a waveguide.
c) Laplace transforms. Properties as for Fourier Transforms. Use in solving initial-value problems for ODEs. Interpretation as half-range Fourier Transforms. Inversion. Examples.
d) Brief tour of other integral transforms, including Mellin, Z, and Radon transforms.

If time permits, at the lecturer's discretion:

x) The delta and Heaviside functions. Their Fourier transforms. Green's functions for linear ODEs. Interpretation of the Fourier transform of the Green's function as the transfer function.
y) Discrete Fourier Transforms and their connection to the continuous transform. Examples, including numerical differentiation by application of FFT.
z) Half-range Fourier transforms. Connection to Laplace transforms. Additive and multiplicative decompositions.

Learning outcomes

By the end of the module, students should be able to:

- Understand the formal definition of asymptotic series and their uses.
- Be able to identify both regular and singular perturbations.
- Understand and be able to use standard techniques to construct asymptotic series for simple perturbation problems involving algebraic equations, integrals, and differential equations.
- Understand and be able to use more advanced techniques to construct asymptotic series in some more complicated perturbation problems, involving matched asymptotic expansions and contour deformation of integrals.
- Be aware of a range of integral transforms and their interpretations.
- Be able to calculate Fourier transforms and their inverses.

- Be aware of the properties of Fourier transforms, and use these properties to solve certain differential and integral equations.
- Be able to calculate Laplace transforms and their inverses.
- Be aware of the properties of Laplace transforms, and use these properties to solve certain differential and integral equations.
- Understand the similarities and differences between Laplace and Fourier transforms.

Subject specific skills

- Ability to apply asymptotics to a wide range of mathematical problems common in maths, physics, and engineering.
- Ability to apply Fourier and Laplace transforms to a wide range of mathematical problems common in maths, physics, and engineering.
- Appreciation of the existence of a variety of other Integral Transforms with potential applied applications.

Transferable skills

Time management. Independent Study. Logical and systematic thinking. Problem solving. Adapting a theoretical framework to real-world problems.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (30%)
Seminars	9 sessions of 1 hour (9%)
Private study	61 hours (61%)
Total	100 hours

Private study description

review course material and exam preparation.

Costs

No further costs have been identified for this module.

Assessment

You must pass all assessment components to pass the module.

Assessment group B1

	Weighting	Study time	Eligible for self-certification
Examination	100%		No
2-hour closed-book exam			

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
Examination	100%		No
2-hour closed-book exam			

- Answerbook Pink (12 page)

Feedback on assessment

Final mark communicated to students at end of year.

Past exam papers for MA269

Availability

Courses

This module is Core option list A for:

- Year 2 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list B for:

- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 2 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- Year 2 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 2 of G100 Mathematics
    - Year 2 of G103 Mathematics (MMath)
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- Year 2 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
- Year 2 of UMAA-G101 Undergraduate Mathematics with Intercalated Year

This module is Option list B for:

- Year 2 of UCSA-G4G1 Undergraduate Discrete Mathematics
- Year 2 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 2 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
    - Year 2 of GF13 Mathematics and Physics
    - Year 2 of FG31 Mathematics and Physics (MMathPhys)
- Year 3 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)

This module is Option list C for:

- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 2 of USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics