MA266-10 Multilinear Algebra

24/25

Department
Warwick Mathematics Institute
Level
Undergraduate Level 2
Module leader
Christian Boehning
Credit value
10
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

It is a second Linear Algebra module, where advanced linear algebra concepts are rigorously developed for students familiar with algebraic tools.

Module aims

It will continue the study of linear algebra, which was begun in Year 1, having benefited from students finishing Abstract Algebra (Algebra-3 or Groups and Rings) in term 1.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

The module is divided in three part:

The first main topic is the Jordan canonical form and related results. Abstractly, this solves the classification problem for pairs (V, T) where V is a finite dimensional vector space over the complex numbers (or any other algebraically closed field) and T a linear self-map of V, up to the equivalence relation induced by bijective linear self-maps of V; more concretely, we classify n by n complex matrices A up to conjugation by invertible matrices P, i.e., the operation A -> P^{-1}AP.

Secondly, we treat bilinear, sesquilinear and quadratic forms on finite dimensional (real and complex) vector spaces. These structures are ubiquitous and fundamental in mathematics and many parts of the sciences. For example, the standard scalar product in R^n is an example. In passing we mention that the description of amplitudes, probabilities and expectation values in quantum theory places such structures at the very heart of how nature works at the smallest levels. We will cover orthonormal basis, Gram-Schmidt process, diagonalisation, singular value decomposition, hermitian forms and normal matrices, among other things.

The third part is concerned with a thorough discussion of the very useful concept of duality (dual vector spaces, dual linear maps, dual bases etc.) and its applications, and after that tensor, exterior and symmetric algebras and their basic properties.

Learning outcomes

By the end of the module, students should be able to:

- develop full command of the theory and computation of the the Jordan canonical form of matrices and linear maps
- learn how to define and to compute functions of matrices
- develop the working knowledge of bilinear forms and quadratic forms
- master the concept of tensor and get proficient manipulating tensors

Indicative reading list

P M Cohn, Algebra, Vol. 1, Wiley, 1982
I N Herstein, Topics in Algebra, Wiley, 1975
Jörg Liesen and Volker Mehrmann, Linear Algebra, Springer, 2015
Peter Petersen, Linear Algebra , Springer, 2012
F. Gantmacher, The Theory of Matrices, American Mathematical Society, 2001
Peter Lax, Linear Algebra and Its Applications, 2nd Edition, Wiley, 2007

View reading list on Talis Aspire

Subject specific skills

This module teaches students to carry out fundamental calculations with matrices, including the theory and computation of the Jordan canonical form of matrices and linear maps; bilinear forms, diagonalizing quadratic forms, and choosing canonical bases for these. After that the module introduces the notion of tensor, treating them rigorously.

Transferable skills

The algorithmic techniques taught have widespread "real world" applications. Examples include ranking in search engines, linear programming and optimisation, signal analysis, and graphics. To also include: clear and precise thinking; the ability to follow complex reasoning; constructing logical arguments, and exposing illogical ones; and formulating problems as algorithms, thereby enhancing understanding of details and rendering them suitable for computer implementation.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (30%)
Seminars	9 sessions of 1 hour (9%)
Private study	61 hours (61%)
Total	100 hours

Private study description

Working on assignments, going over lecture notes, text books, exam revision.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D1

	Weighting	Study time
Assignments	15%	
Examination	85%	

- Answerbook Pink (12 page)

Assessment group R1

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Pink (12 page)

Feedback on assessment

Marked homework (both assessed and formative) is returned and discussed in smaller classes.

Exam feedback is given.

Past exam papers for MA266

Availability

Courses

This module is Core for:

- Year 2 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 2 of G103 Mathematics (MMath)
    - Year 2 of G103 Mathematics (MMath)
- UMAA-GV17 Undergraduate Mathematics and Philosophy
    - Year 2 of GV17 Mathematics and Philosophy
    - Year 2 of GV17 Mathematics and Philosophy
    - Year 2 of GV17 Mathematics and Philosophy
- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core optional for:

- UMAA-G100 Undergraduate Mathematics (BSc)
    - Year 2 of G100 Mathematics
    - Year 2 of G100 Mathematics
    - Year 2 of G100 Mathematics
- Year 2 of UMAA-G103 Undergraduate Mathematics (MMath)
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-G101 Undergraduate Mathematics with Intercalated Year

This module is Option list A for:

- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
    - Year 2 of GF13 Mathematics and Physics
    - Year 2 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
    - Year 2 of FG31 Mathematics and Physics (MMathPhys)
    - Year 2 of FG31 Mathematics and Physics (MMathPhys)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
    - Year 2 of GG14 Mathematics and Statistics
    - Year 2 of GG14 Mathematics and Statistics

This module is Option list B for:

- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 2 of G4G1 Discrete Mathematics
  - Year 2 of G4G1 Discrete Mathematics
- Year 2 of UCSA-G4G3 Undergraduate Discrete Mathematics
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
  - Year 2 of Y602 Mathematics,Operational Research,Stats,Economics
  - Year 2 of Y602 Mathematics,Operational Research,Stats,Economics