# ES98A-15 Fundamentals of Predictive Modelling

### 24/25

Department School of Engineering Level Taught Postgraduate Level Module leader Tim Sullivan Credit value 15 Module duration 10 weeks Assessment 60% coursework, 40% exam Study location University of Warwick main campus, Coventry

# Description

### Introductory description

This module provides students with fundamental knowledge for predictive modelling and uncertainty quantification. It gives an overview of the essential elements of the mathematical, statistical, and computational techniques needed to provide well-calibrated predictions for the behaviour of physical systems.

### Module aims

Understand how to use statistical modelling to quantify uncertainty arising from computer simulation and epistemic and aleatoric uncertainty in scientific problems. Formulate and solve Bayesian inverse problems of high-dimensional quantities of interest. Make well-calibrated predictions for the behaviour of physical systems described by mathematical models and data.

### **Outline syllabus**

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- Univariate probability pre-reading discrete and continuous distributions, the Law of Large Numbers, expectations and variance
- Multivariate probability
- Bayesian probability
- Multivariate Gaussians and their conditioning
- State estimation via Kalman filtering
- Forward and inverse UQ
- Quadrature and sampling (MC, MCMC, QMC, LHS, ...)
- Finite-dimensional linear algebra
- Basic functional analysis (infinite-dimensional linear algebra)
- Perspectives on random functions
- Interpolation, regression, orthogonal projection
- Motivation for regularisation
- Tikhonov regularisation / ridge regression
- Sparse approximation (\ell^1 regularisation)
- Optimisation approach to inverse problems, linear and nonlinear case, connect to Tikhonov regularisation
- · Bayesian approach to inverse problems, well-posedness of BIPs
- High- or infinite-dimensional aspects
- Approximate Bayesian inference e.g. Laplace approximation,

#### Learning outcomes

By the end of the module, students should be able to:

- Understand and apply univariate and multivariate probability to complex scientific modelling problems
- Understand and interpret forward and inverse uncertainty quantification
- Apply and evaluate quadrature and sampling schemes to solve predictive modelling tasks
- Appreciate, synthesise, and apply modelling strategies for uncertain quantities of interest in high dimension
- Contextualise the relations among regression, approximation, and orthogonal projection
- Recognise, formulate, analyse and solve Bayesian inverse problems

#### Indicative reading list

McClarren, Uncertainty Quantification and Predictive Computational Science, available from SpringerLink on campus at Warwick - aimed at a general Physical Sciences / Engineering audience

Sullivan, Introduction to Uncertainty Quantification, available from SpringerLink on campus at Warwick

Boyd and Vandenberghe, Introduction to Applied Linear Algebra - Vectors, Matrices and Least Squares, freely available online. Companion exercises that implement the material in Python and Julia are available from the same webpage.

### Interdisciplinary

The MSc programme will recruit students with backgrounds across the physical and mathematical sciences, including engineering, and will provide an interdisciplinary perspective on predictive modelling.

The fundamentals of uncertainty quantification and predictive modelling are drawn from the mathematical, statistical and computational sciences, and rely on disciplinary fusion with application domains such as physical/chemical science and engineering.

### Subject specific skills

- Computational statistics
- Mathematical modelling
- Predictive modelling
- Reliability assessment of descriptive and predictive scientific models

### Transferable skills

- Data analysis and modelling
- Oral presentation skills
- Scientific computing

# Study

# Study time

Туре	Required
Lectures	20 sessions of 1 hour (13%)
Seminars	10 sessions of 1 hour (7%)
Supervised practical classes	10 sessions of 1 hour (7%)
Other activity	10 hours (7%)
Private study	100 hours (67%)
Total	150 hours

### Private study description

NB:

- Lectures and faculty-taught lab sessions take place in weeks 1-10 of module. 2x1h (or 1x2h) lecture per week plus 1x1h lab session ("supervised practical") per week.
- GTA-taught lab sessions ("seminars") take place in weeks 1-10 of module: 1x1h lab session per week.

- Students attend WCPM seminar in weeks 1-10 of module, i.e. throughout the teaching term
- Viva takes place in week 11, referring to all taught components, preliminary work on MSc research project, and WCPM seminars attended

#### Other activity description

Attendance at weekly WCPM seminars, to gain an appreciation of predictive modelling topics as they arise in research and practice.

### Costs

No further costs have been identified for this module.

### Assessment

You must pass all assessment components to pass the module.

### Assessment group D1

	Weighting	Study time	
Computer Laboratory Assignments	60%		
5 x ono-pago assignments based upon	locture topic and computer lab	oratory work	$( \cap$

40%

5 x one-page assignments based upon lecture topic and computer laboratory work. (One additional formative assignment will correspond to "week 0" induction material, and the others to the taught content of the module.)

Oral Examination

- 1. Student presentation of how predictive modelling and uncertainty quantification are relevant to their chosen MSc research project. To be answered with reference to module content, initial scoping of research project, and relevant seminars.
- 2. Unseen questions on PM and UQ, based on a pre-circulated list of general topics aligned with the lectures and laboratory sessions

#### Feedback on assessment

- Annotation of computer workbooks / PDFs with feedback on individual questions, or equivalent individual written feedback.
- Written feedback from examiners of viva voce exam.

Past exam papers for ES98A

# Availability

### Courses

This module is Core for:

• Year 1 of TESA-H1B1 Postgraduate Taught Predictive Modelling and Scientific Computing