ST342-15 Mathematics of Random Events

23/24

Department
Statistics
Level
Undergraduate Level 3
Module leader
Karen Habermann
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

This module runs in Term 1 and aims to provide an introduction to this theory, concentrating on examples and applications. This course would particularly be useful for students willing to learn more about probability theory, analysis, mathematical finance, and theoretical statistics.

This module is available for students on a course where it is an optional core module or listed option and as an Unusual Option to students who have completed the prerequisite modules.

Pre-requisites

- Statistics Students: ST218 Mathematical Statistics A AND ST219 Mathematical Statistics B
- Non-Statistics Students: ST220 Introduction to Mathematical Statistics

Leads to ST318 Probability Theory.

Module web page

Module aims

To introduce the concepts of measurable spaces, integral with respect to the Lebesgue measure, independence and modes of convergence, and provide a basis for further studies in Probability, Statistics and Applied Mathematics. Imagine picking a real number x between 0 and 1 "at random" and with perfect accuracy, so that the probability that this number belongs to any interval within [0,1] is equal to the length of the interval. Can we compute the probability of x belonging to any subset to [0,1]?
To answer this question rigorously we need to develop a mathematical framework in which we can model the notion of picking a real number "at random". The mathematics we need, called measure theory, permeates through much of modern mathematics, probability and statistics.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Algebras, sigma-algebras and measures
   Algebra and contents, sigma-algebra and measures, pi-systems, examples of random events and measurable sets.

2. Lebesgue integration
   Simple functions, standard representations, measurable functions, Lebesgue integral, properties of integrals, integration of Borel functions.

3. Product measures
   Sections, product sigma-algebras, product measures, Fubini theorem.

4. Independence and conditional expectation
   Independence of sigma-algebras, independence of random variables, conditional expectation with respect to a simple algebra.

5. Convergence and modes of convergence
   Borel-Cantelli lemma, Fatou lemma, dominated convergence theorem, modes of convergence of random variables, Markov inequality and application, weak and strong laws of large numbers.

Learning outcomes

By the end of the module, students should be able to:

- Explain the properties of the probability spaces one can use for building models for simple experiments.
- Compute the probabilities of complicated events using countable additivity.
- Properly formulate the notion of statistical independence.
- Describe the basic theory of integration, particularly as applied to the expectation of random variables, and be able to compute expectations from first principles.
- Identify convergence in probability and almost sure convergence of sequences of random variables, and use and justify convergence in the computation of integrals and expectations.

Indicative reading list

View reading list on Talis Aspire

Subject specific skills

- Demonstrate facility with rigorousprobabilistic methods.
- Evaluate, select and apply appropriate mathematical and/or probabilist techniques.
- Demonstrate knowledge of and facility with formal probability concepts, both explicitly and by applying them to the solution of mathematical problems.
- Create structured and coherent arguments communicating them in written form.
- Construct logical mathematical arguments with clear identification of assumptions and conclusions.
- Reason critically, carefully, and logically and derive (prove) mathematical results.

Transferable skills

- Problem solving: Use rational and logical reasoning to deduce appropriate and well-reasoned conclusions. Retain an open mind, optimistic of finding solutions, thinking laterally and creatively to look beyond the obvious. Know how to learn from failure.
- Self awareness: Reflect on learning, seeking feedback on and evaluating personal practices, strengths and opportunities for personal growth.
- Communication: Present arguments, knowledge and ideas, in a range of formats.
- Professionalism: Prepared to operate autonomously. Aware of how to be efficient and resilient. Manage priorities and time. Self-motivated, setting and achieving goals, prioritising tasks.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	5 sessions of 1 hour (3%)
Private study	115 hours (77%)
Total	150 hours

Private study description

Weekly revision of lecture notes and materials, wider reading, practice exercises and preparing for examination.

Costs

No further costs have been identified for this module.

Assessment

You must pass all assessment components to pass the module.

Assessment group B4

	Weighting	Study time	Eligible for self-certification
In-person Examination	100%		No

The examination paper will contain four questions, of which the best marks of THREE questions will be used to calculate your grade.

- Answerbook Pink (12 page)

Assessment group R3

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

The examination paper will contain four questions, of which the best marks of THREE questions will be used to calculate your grade.

- Answerbook Pink (12 page)

Feedback on assessment

Solutions and cohort level feedback will be provided for the examination. The results of the January examination will be available by the end of week 10 of term 2.

Past exam papers for ST342

Availability

Anti-requisite modules

If you take this module, you cannot also take:

- MA359-15 Measure Theory

Courses

This module is Core optional for:

- Year 3 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
    - Year 3 of G30E Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream) Int
    - Year 4 of G30E Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream) Int

This module is Optional for:

- Year 3 of UCSA-G4G1 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics

This module is Core option list B for:

- Year 3 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
    - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Option list A for:

- Year 3 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)

- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)
- Year 3 of USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics

This module is Option list C for:

- Year 3 of USTA-G302 Undergraduate Data Science
- Year 3 of USTA-G304 Undergraduate Data Science (MSci)
- Year 4 of USTA-G303 Undergraduate Data Science (with Intercalated Year)