# ST339-15 Introduction to Mathematical Finance

## 23/24

## **Department**

**Statistics** 

Level

**Undergraduate Level 3** 

Module leader

**David Hobson** 

Credit value

15

**Module duration** 

10 weeks

**Assessment** 

Multiple

**Study location** 

University of Warwick main campus, Coventry

# **Description**

# Introductory description

This module runs is available for students on a course where it is a listed option and as an Unusual Option to students who have completed the prerequisite modules.

#### **Pre-requisites**

- Statistics Students:
  - ST218 Mathematical Statistics A; or,
  - ST228 Mathematical Methods for Statistics and Probability and ST229 Probability for Mathematical Statistic.
- Non-Statistics Students:
  - ST220/ST226 Introduction to Mathematical Statistics; or,
  - ST232/ST233 Introduction to Mathematical Statistics.

It is **strongly recommended** to take either MA359 Measure theory or ST342 Mathematics of Random Events/ST3xx Measure Theory for Probability alongside this module.

Results from this module will be partly used to determine exemption eligibility in the Institute and

Faculty of Actuaries (IFoA) module CM2.

This module serves as a prerequisite for ST401 Stochastic Methods in Finance, IB357 Investment Management

IB359 Derivatives and Risk Management, IB394 International Financial Management and EC334 Topics in Financial Economics: Corporate Finance and Markets.

#### Module web page

#### Module aims

To provide an introduction to Mathematical Finance in discrete time and cover the discrete part of the actuarial syllabus.

To be able to evaluate and interpret the theory of mathematical finance in discrete time and to apply theoretical concepts to construct stochastic models of financial markets.

# **Outline syllabus**

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- 1. No-Arbitrage and the Fundamental Theorem of Asset Pricing
  - a) Mathematical model for one period financial markets
  - b) Trading strategies and arbitrage opportunities
  - c) Discounting and Equivalent Martingale Measures
  - d) The Fundamental Theorem of Asset Pricing
- 2. Mean-Variance Portfolio Selection and the CAPM
  - a) The return of an asset and of a portfolio
  - b) Maximising the expected return
  - c) The mean-variance problems
  - d) The case without a riskless asset
  - e) The case with a riskless asset
  - f) The Markowitz tangency portfolio and the capital market line
  - g) Mean-variance equilibria
  - h) The Capital Asset Pricing Model (CAPM)
- 3. Utility Theory
  - a) Preferences on lotteries
  - b) Von Neumann-Morgenstern representation
  - c) Concave functions and Jensen's inequality
  - d) Expected utility representation
  - e) Measuring risk aversion
  - d) A primer on utility maximisation
- 4. Introduction to Risk Measures
  - a) Monetary measures of risk
  - b) Value at Risk and Expected Shortfall
- 5. Pricing and Hedging in Finite Discrete Time
  - a) Conditional expectations

- b) Filtrations and martingales
- c) Financial markets in finite discrete time
- d) Self-financing strategies
- e) The Fundamental Theorem of Asset Pricing revisited
- f) Valuation of contingent claims
- g) Complete markets
- h) Pricing and hedging in the binomial model

## Learning outcomes

By the end of the module, students should be able to:

- Understand key notions of arbitrage and equivalent martingale measures in a one period financial market; calculate the set of equivalent martingale measures in a financial market
- Solve the mean-variance problems, understand the concept of the capital market line, describe the Capital Asset Pricing model including the principal results and assumptions
- Describe preference orders of financial investors, explain the concept of risk aversion, solve simple utility maximisation problems
- Explain the modern concept of monetary measures of risk, Calculate the Value at Risk and Expected Shortfall for given distributions
- Model financial markets in finite discrete time, describe self-financing strategies and absence
  of arbitrage, hedge derivate products in complete and incomplete markets in discrete time

# Indicative reading list

H. Föllmer and A. Schied: Stochastic Finance. An Introduction in Discrete Time, 4th ed., de Gruyter, 2016.

S.F. LeRoy and J. Werner: Principles of Financial Economics, 2nd ed., Cambridge University Press, 2014.

S.E. Shreve: Stochastic Calculus for Finance 1: The Binomial Asset Pricing Model, Springer, 2003.

J. Jacod and P. Protter: Probability Essentials, Springer, 2003.

View reading list on Talis Aspire

# Subject specific skills

- Demonstrate facility with rigorousprobabilistic methods.
- Evaluate, select and apply appropriate mathematical and/or probabilist techniques.
- Demonstrate knowledge of and facility with formal probability concepts, both explicitly and by applying them to the solution of finance problems.
- Create structured and coherent arguments communicating them in written form.
- Construct logical mathematical arguments with clear identification of assumptions and conclusions.

• Reason critically, carefully, and logically and derive (prove) mathematical results.

#### Transferable skills

- Problem solving: Use rational and logical reasoning to deduce appropriate and wellreasoned conclusions. Retain an open mind, optimistic of finding solutions, thinking laterally and creatively to look beyond the obvious. Know how to learn from failure.
- Self awareness: Reflect on learning, seeking feedback on and evaluating personal practices, strengths and opportunities for personal growth.
- Communication: Present arguments, knowledge and ideas, in a range of formats.
- Professionalism: Prepared to operate autonomously. Aware of how to be efficient and resilient. Manage priorities and time. Self-motivated, setting and achieving goals, prioritising tasks.

# **Study**

# Study time

Туре	Required	Optional
Lectures	30 sessions of 1 hour (20%)	2 sessions of 1 hour
Tutorials	5 sessions of 1 hour (3%)	
Private study	115 hours (77%)	
Total	150 hours	

# **Private study description**

Weekly revision of lecture notes and materials, wider reading, practice exercises and preparing for examination.

## Costs

No further costs have been identified for this module.

#### **Assessment**

You must pass all assessment components to pass the module.

## **Assessment group B4**

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Study time

In-person Examination

100%

The examination paper will contain four questions, of which the best marks of THREE questions will be used to calculate your grade.

Answerbook Pink (12 page)

# **Assessment group R3**

Weighting

Study time

In-person Examination - Resit

100%

The examination paper will contain four questions, of which the best marks of THREE questions will be used to calculate your grade.

Answerbook Pink (12 page)

#### Feedback on assessment

Solutions and cohort level feedback will be provided for the examination.

Past exam papers for ST339

# **Availability**

# **Anti-requisite modules**

If you take this module, you cannot also take:

- EC333-15 Topics in Financial Economics: Theories and International Finance
- IB253-12 Principles of Finance 1
- IB253-15 Principles of Finance 1

## Courses

This module is Core for:

- Year 3 of USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics (with Intercalated

- Year 3 of G30E Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream) Int
- Year 4 of G30E Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream) Int

#### This module is Optional for:

- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
  - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics

#### This module is Option list A for:

- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
- Year 3 of UMAA-G103 Undergraduate Mathematics (MMath)
- USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
  - Year 3 of G1G3 Mathematics and Statistics (BSc MMathStat)
  - Year 4 of G1G3 Mathematics and Statistics (BSc MMathStat)
- USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
  - Year 4 of G1G4 Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
  - Year 5 of G1G4 Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 3 of GG14 Mathematics and Statistics
  - Year 3 of GG14 Mathematics and Statistics
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics, Operational Research, Statistics, Economics (with Intercalated Year)

#### This module is Option list B for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe

- Year 3 of G106 Mathematics (MMath) with Study in Europe
- Year 4 of G106 Mathematics (MMath) with Study in Europe

# This module is Option list C for:

- USTA-G302 Undergraduate Data Science
  - Year 3 of G302 Data Science
  - Year 3 of G302 Data Science
- Year 3 of USTA-G304 Undergraduate Data Science (MSci)
- Year 4 of USTA-G303 Undergraduate Data Science (with Intercalated Year)