ST333-15 Applied Stochastic Processes

23/24

Department
Statistics
Level
Undergraduate Level 3
Module leader
Sam Olesker-Taylor
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

This module runs is available for students on a course where it is a listed option and as an Unusual Option to students who have completed the prerequisite modules.

The ideas presented in this module have a vast range of applications, for example routing algorithms in telecommunications (queues), assessment of apparent spatial order in astronomical data (stochastic geometry), description of outbreaks of disease (epidemics). We will only be able to introduce each area - indeed each area could easily be the subject of a course on its own! But the introduction will provide you with a good base to follow up where and when required. (For example: a MORSE graduate found that their firm was asking them to address problems in queuing theory, for which ST333 provided the basis.) We will discuss these and other applications and show how the ideas of stochastic process theory help in formulating and solving relevant questions.

Pre-requisites: ST202 Stochastic Processes or ST227 Stochastic Processes.

Results from this module may be partly used to determine of exemption eligibility in the Institute and Faculty of Actuaries (IFoA) modules CS2.

Module web page

Module aims

To provide an introduction to concepts and techniques which are fundamental in modern applied probability theory and operations research:
Models for queues, point processes, and epidemics.
Notions of equilibrium, threshold behaviour, and description of structure.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Continuous time Markov Chains.
   Terms used in the analysis of continuous-time Markov chains: Markov property, transition probability function, standing assumptions, Chapman-Kolmogorov equations, Q-matrix, Kolmogorov forward and backward differential equations, equilibrium distribution. The simplest case: finite state-space Markov chains. The "switcher" example. Exact transition densities for processes on a small number of states. The strong Markov property.

2. Linear Birth-Death processes.
   Poisson (counting) process: construction, ideas of independent increments, superposition, counts and thinning. Pure birth process, linear birth-death process, birth-death-immigration process: construction using "microscopic model", derivation of extinction and equilibrium probabilities. Generalized birth-death processes.

3. Queuing theory.
   The Markov single-server (M/M/1) queue. The concept of detailed balance. Measures of effectiveness. Multiserver (M/M/cl/c2) queues. Erlang's formula. Queues with general service-time distribution (M/G/l) and their embedded Markov chains. Little's formula, Pollaczek-Khintchine formula.

4. Other Markov properties.
   Stopping times. Strong Markov property. Holding theorem.

5. Epidemics.
   Deterministic Epidemic model. Stochastic model without removals. Stochastic model with removals.

Learning outcomes

By the end of the module, students should be able to:

- formulate continuous-time Markov chain models for applied problems.
- use basic theory to gain quick answers to important questions (for example, what is the equilibrium distribution for a specific reversible Markov chain?).
- solve for the transition probabilities for Markov chains on a finite state space.
- understand how to use Markov chains in the modelling and analysis of queues and epidemics.

Indicative reading list

View reading list on Talis Aspire

Subject specific skills

- Demonstrate facility with rigorousprobabilistic methods.
- Evaluate, select and apply appropriate mathematical and/or probabilist techniques.
- Demonstrate knowledge of and facility with formal probability concepts, both explicitly and by applying them to the solution of mathematical problems.
- Create structured and coherent arguments communicating them in written form.
- Construct logical arguments with clear identification of assumptions and conclusions.
- Reason critically, carefully, and logically.

Transferable skills

- Problem solving: Use rational and logical reasoning to deduce appropriate and well-reasoned conclusions. Retain an open mind, optimistic of finding solutions, thinking laterally and creatively to look beyond the obvious. Know how to learn from failure.
- Self awareness: Reflect on learning, seeking feedback on and evaluating personal practices, strengths and opportunities for personal growth.
- Communication: Present arguments, knowledge and ideas, in a range of formats.
- Professionalism: Prepared to operate autonomously. Aware of how to be efficient and resilient. Manage priorities and time. Self-motivated, setting and achieving goals, prioritising tasks.

Study

Study time

Type	Required	Optional
Lectures	30 sessions of 1 hour (20%)	2 sessions of 1 hour
Seminars	5 sessions of 1 hour (3%)	
Private study	115 hours (77%)	
Total	150 hours	

Private study description

Completion of non-credit bearing coursework, weekly revision of lecture notes and materials, wider reading, practice exercises and preparing for examination.

Costs

No further costs have been identified for this module.

Assessment

You must pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group B4

	Weighting	Study time
In-person Examination	100%	

The examination paper will contain four questions, of which the best marks of THREE questions will be used to calculate your grade.

- Answerbook Pink (12 page)
- Students may use a calculator

Assessment group R3

	Weighting	Study time
In-person Examination - Resit	100%	

The examination paper will contain four questions, of which the best marks of THREE questions will be used to calculate your grade.

- Answerbook Pink (12 page)
- Students may use a calculator

Feedback on assessment

Opportunities will be provided to submit non-credit bearing coursework for which feedback will be provided in the following problem class.
Solutions and cohort level feedback will be provided for the examination.

Past exam papers for ST333

Availability

Anti-requisite modules

If you take this module, you cannot also take:

- ST406-15 Applied Stochastic Processes with Advanced Topics

Courses

This module is Optional for:

- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
  - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
  - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics

This module is Unusual option for:

- Year 2 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)

This module is Core option list A for:

- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Option list A for:

- USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
  - Year 3 of G1G3 Mathematics and Statistics (BSc MMathStat)
  - Year 4 of G1G3 Mathematics and Statistics (BSc MMathStat)
- USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
  - Year 4 of G1G4 Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
  - Year 5 of G1G4 Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 3 of GG14 Mathematics and Statistics
  - Year 3 of GG14 Mathematics and Statistics
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
  - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
  - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics,Operational Research,Statistics,Economics (with Intercalated Year)

This module is Option list B for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- Year 3 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
  - Year 3 of G30E Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream) Int
  - Year 4 of G30E Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream) Int
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
  - Year 4 of G106 Mathematics (MMath) with Study in Europe
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year

This module is Option list C for:

- USTA-G302 Undergraduate Data Science
  - Year 3 of G302 Data Science
  - Year 3 of G302 Data Science
- Year 3 of USTA-G304 Undergraduate Data Science (MSci)
- Year 4 of USTA-G303 Undergraduate Data Science (with Intercalated Year)

This module is Option list F for:

- Year 3 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated

- Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)