PX285-15 Hamiltonian and Fluid Mechanics

23/24

Department
Physics
Level
Undergraduate Level 2
Module leader
Valery Nakariakov
Credit value
15
Module duration
20 weeks
Assessment
100% exam
Study location
University of Warwick main campus, Coventry

Description

Introductory description

This module looks at the Hamiltonian and Lagrangian formulation of classical mechanics and introduces the mechanics of fluids. Lagrangian and Hamiltonian mechanics have provided the natural framework for several important developments in theoretical physics including quantum mechanics.

The field of fluids is one of the richest and most easily appreciated in physics. Tidal waves, cloud formation and the weather generally are some of the more spectacular phenomena encountered in fluids. The module establishes the basic equations of motion for a fluid - the Navier-Stokes equations - and shows that in many cases they can yield simple and intuitively appealing explanations of fluid flows.

Module web page

Module aims

To revise the key elements of Newtonian mechanics and use this to develop Lagrangian and Hamiltonian mechanics. The module should also explain why PDEs (with associated boundary conditions) are an appropriate model for fluids. The module should prepares students for future applied mathematics modules.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Hamiltonian and Lagrangian Mechanics: Analogy with optics and constructive interference; principle of least action; examples of Euler Lagrange Equations. 1-D trajectory, T-V case, worked examples; T+V as a constant of the motion; multiple coordinates with examples. Generalised coordinates and canonical momenta. Polar coordinates; angular momentum; moment of inertia of rigid bodies; treatment of constraints. Symmetry and Conservation Laws. Hamiltonian formulation. Hamilton's equations, phase space, examples. Normal Modes and Small Oscillations. Inertial and stiffness matrices, diatomic and triatomic molecules.

Fluids: Materials which do not support shear. Idea of a Newtonian fluid. Plausibility of $\tau = \mu \, \partial u/\partial y$ from assumption of a relaxation time for stress. Hydrostatics, forces due to pressure and gravity. Hydrodynamics: acceleration, continuity and incompressibility. Euler equation. Streamlines: Integrating Euler for steady flow along a streamline to give Bernoulli. Energy considerations. Applications of Bernoulli: flux through a hole, Pitot-static tube, aerofoil, waves on shallow water. Hydrodynamics of Viscous Flow: Forces due to viscosity, Navier-Stokes equation. Poiseuille formula for laminar flow between plates. Turbulence, role of Reynolds number. Physical interpretation of Re as Inertial forces/Viscous forces. Irrotational Flow: Definition of vorticity and circulation, Kelvin's circulation theorem. Uniform flow, flow past a cylinder. Lift on thin aerofoil, as example for Magnus Effect. Circulation around a cylinder. Vortices. Advection of unlike vortices. The vortex ring. Circling of like vortices. Vortices at edges of wings. Real Flows: Idea of boundary layer; Boundary layer separation and drag crisis.

Learning outcomes

By the end of the module, students should be able to:

- Derive and solve the Euler-Lagrange equations for simple models
- Construct the Hamiltonian function, and derive and solve Hamilton's equations for simple systems
- Explain the role of (and relations between) constraints, conserved quantities and generalised coordinates
- Use dimensional analysis to analyse fluid flows. In particular, students should appreciate the relevance of the Reynolds number and the boundary layer concept.
- Be familiar with the equations of motion for incompressible fluids (the Navier- Stokes equations) and associated boundary conditions
- Simplify the equations of motion in the case of incompressible irrotational flow and solve them for simple cases including vortices

Indicative reading list

H Goldstein, Classical Mechanics; A helpful reference for the beginning of the module is: Feynmann, Leighton & Sands, The Feynmann Lectures on Physics, Vol 2, Chapter 19
LD Landau and EM Lifshitz, Fluid Mechanics, Pergamon; DJ Tritton, Physical Fluid Dynamics, OUP; TE Faber Fluid Dynamics for Physicists, CUP

View reading list on Talis Aspire

Interdisciplinary

The Hamiltonian and Lagrangian formulations of mechanics use ideas from mathematics - variational methods and symplectic structures, while the study of fluids has always crossed the boundaries between mathematics, physics and engineering. This module, taken by large numbers of physics, maths and maths/physics students, introduces the discipline from a physicist's perspective. It leads on to modules taught in later years by Maths.

Subject specific skills

Knowledge of mathematics and physics. Skills in modelling, reasoning, thinking.

Transferable skills

Analytical, communication, problem-solving, self-study

Study

Study time

Type	Required
Lectures	40 sessions of 1 hour (27%)
Private study	110 hours (73%)
Total	150 hours

Private study description

Working through lecture notes, solving problems, wider reading, discussing with others taking the module, revising for the exam, practising on past exam papers

Costs

No further costs have been identified for this module.

Assessment

You must pass all assessment components to pass the module.

Assessment group B

	Weighting	Study time	Eligible for self-certification
Assessment component			
2 hour examination Answer 4 questions	100%		No

- Answerbook Pink (12 page)
- Students may use a calculator

Reassessment component is the same

Feedback on assessment

Personal tutor, group feedback

Past exam papers for PX285

Availability

Courses

This module is Core for:

- Year 2 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 2 of GF13 Mathematics and Physics
  - Year 2 of FG31 Mathematics and Physics (MMathPhys)

This module is Option list A for:

- Year 2 of UPXA-F300 Undergraduate Physics (BSc)
- UPXA-F303 Undergraduate Physics (MPhys)
  - Year 2 of F300 Physics
  - Year 2 of F303 Physics (MPhys)
- Year 2 of UPXA-F3F5 Undergraduate Physics with Astrophysics (BSc)
- Year 2 of UPXA-F3FA Undergraduate Physics with Astrophysics (MPhys)

This module is Option list B for:

- Year 2 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- Year 2 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 2 of G100 Mathematics
    - Year 2 of G103 Mathematics (MMath)
- Year 2 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- Year 2 of UMAA-G101 Undergraduate Mathematics with Intercalated Year