

PH342-15 Philosophy of Mathematics

23/24

Department

Philosophy

Level

Undergraduate Level 3

Module leader

Benedict Eastaugh

Credit value

15

Module duration

10 weeks

Assessment

20% coursework, 80% exam

Study location

University of Warwick main campus, Coventry

Description

Introductory description

Do mathematical objects such as numbers and sets exist or are they merely useful fictions? What is the nature of mathematical knowledge and how is it distinct from our knowledge of the physical world? What, if any, is the connection between the two? What role does mathematics play in the empirical sciences? What is the correct logic for reasoning about mathematics? Are formally undecidable statements (e.g. the Parallel Postulate, the Gödel sentence, the Continuum Hypothesis) objectively true or false? This module will explore different ways in which philosophy might be of help in answering these questions, both from the contemporary perspective and that of the major foundational schools of the late 19th and early 20th centuries: logicism, intuitionism, and formalism.

Module aims

This module has two goals: 1) to familiarise students with major developments in the foundations of mathematics from the late 19th century onward; and 2) to illustrate how these developments inform contemporary debates in philosophy of mathematics.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be

covered. Actual sessions held may differ.

Week 1: Introduction to the module. The infinite in ancient Greek thought.

- Core reading: introduction and chapters 1 and 2 of A. W. Moore, *The Infinite* (Routledge, 2001).

Week 2: Infinity in mathematics. Cantor's theory of sets.

- Core reading: part I (pages 1–34) of M. Giaquinto, *The Search for Certainty* (OUP, 2002).

Week 3: The class-theoretic paradoxes. Type theory and limitation of size.

– Core reading: chapter 10 of Moore (2001), part II (pages 35–65) of Giaquinto (2002).

Week 4: The axiomatic method. Hilbertian finitism.

– Core reading: Chapters IV.3–4 of Giaquinto (2002), D. Hilbert, *On the infinite* (1926).

Week 5: Constructivism in Brouwer and Heyting.

– Core reading: chapter 7 of S. Shapiro, *Thinking About Mathematics* (OUP, 2000), introduction to Brouwer's Cambridge lectures on intuitionism (CUP, 1981).

Week 7: The Löwenheim–Skolem theorem. Skolem's paradox.

– Core reading: Chapters IV.1 and IV.2 of Giaquinto (2002), P. Benacerraf, *Skolem and the skeptic* (1985).

Week 8: The continuum problem. Realism and indeterminacy in set theory.

– Core reading: Chapter VI.1 of Giaquinto (2002), K. Gödel, *What is Cantor's continuum problem?* (1947).

Week 9: Categoricity and determinacy. Structuralism.

– Core reading: Chapter 10 of Shapiro (2000), P. Benacerraf, *What numbers could not be* (1965).

Week 10: Potential infinity revisited. Modality and potentiality.

– Core reading: Linnebo and Shapiro, *Actual and potential infinity* (2019).

Learning outcomes

By the end of the module, students should be able to:

- Demonstrate knowledge of some of the central topics in the philosophy of mathematics, and of the historical development of key approaches to the philosophy of mathematics (Subject knowledge and understanding)
- Understand the significance that questions in the philosophy of mathematics have to wider issues in philosophy and the foundations of mathematics (cognitive skills)
- Articulate their own view of the relative merits of different theories and engage critically with the arguments put forward in support of them (key skills)
- Show an understanding of methodological issues in the philosophy of mathematics, and of questions of demarcation between philosophy and mathematics (subject-specific skills)

Indicative reading list

Much of the background and historical reading will be drawn from two books:

The Infinite (2nd ed.) by A. W. Moore (Routledge, 2001).

The Search for Certainty by M. Giaquinto (Oxford University Press, 2002).

There will also be a substantial use of original sources and recent scholarship. Many important papers can be found in the following collections.

Philosophy of Mathematics: Selected Readings (2nd ed.), edited by P. Benacerraf and H. Putnam (Cambridge University Press, 1983).

From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931, edited by J. van Heijenoort.

An excellent handbook in philosophy of mathematics is:

The Oxford Handbook of Philosophy of Mathematics and Logic, edited by S. Shapiro (Oxford University Press, 2005).

A survey of topics in mathematical logic that are relevant for this course is:

Mathematical Logic by J. R. Shoenfield (Addison-Wesley, 1967).

Subject specific skills

Show an understanding of methodological issues in the philosophy of mathematics, and of questions of demarcation between philosophy and mathematics.

Transferable skills

Understand how major debates in the philosophy of mathematics—e.g. between logicism, formalism, and intuitionism—are related to topics in the history of philosophy, metaphysics, and epistemology. Appreciate how developments in mathematical logic—e.g. axiomatic set theory, proof theory—grew out of concern for foundational issues in the 19th and early 20th century.

Study

Study time

Type	Required
Lectures	9 sessions of 2 hours (12%)
Seminars	8 sessions of 1 hour (5%)
Private study	124 hours (83%)
Total	150 hours

Private study description

No private study requirements defined for this module.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group DA

	Weighting	Study time
1000 word essay	20%	
Online Examination	80%	
2 hour exam		
~Platforms - AEP		

- Online examination: No Answerbook required

Feedback on assessment

Written feedback on essays and exams.

[Past exam papers for PH342](#)

Availability

Pre-requisites

The module is designed to be as self-contained as possible. However, you should be aware that several of the topics we will discuss are related to developments in mathematical logic (as treated in modules like PH210 Logic 2, PH340 Logic 3 and MA3H3 Set Theory), and also build on philosophical themes which are covered in modules like PH251 Metaphysics, PH252 Epistemology, and PH144 Mind and Reality. Background in these subjects will therefore be helpful for fully engaging with the module content.

Courses

This module is Core for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Optional for:

- UPHA-L1CA Undergraduate Economics, Psychology and Philosophy
 - Year 2 of L1CA Economics, Psychology and Philosophy
 - Year 2 of L1CC Economics, Psychology and Philosophy (Behavioural Economics Pathway)
 - Year 2 of L1CD Economics, Psychology and Philosophy (Economics with Philosophy Pathway)
 - Year 2 of L1CE Economics, Psychology and Philosophy (Philosophy and Psychology Pathway)
 - Year 3 of L1CA Economics, Psychology and Philosophy
 - Year 3 of L1CC Economics, Psychology and Philosophy (Behavioural Economics Pathway)
 - Year 3 of L1CD Economics, Psychology and Philosophy (Economics with Philosophy Pathway)
 - Year 3 of L1CE Economics, Psychology and Philosophy (Philosophy and Psychology Pathway)
- UPHA-L1CB Undergraduate Economics, Psychology and Philosophy (with Intercalated Year)
 - Year 4 of L1CG Economics, Psychology and Philosophy (Behavioural Economics Pathway) (with Intercalated Year)
 - Year 4 of L1CH Economics, Psychology and Philosophy (Economics with Philosophy Pathway) (with Intercalated Year)
 - Year 4 of L1CJ Economics, Psychology and Philosophy (Philosophy and Psychology Pathway) (with Intercalated Year)
 - Year 4 of L1CB Economics, Psychology and Philosophy (with Intercalated Year)
 - Year 4 of L1CB Economics, Psychology and Philosophy (with Intercalated Year)
- UHIA-V1V8 Undergraduate History and Philosophy (with Year Abroad and a term in Venice)
 - Year 3 of V1V8 History and Philosophy (with Year Abroad and a term in Venice)
 - Year 4 of V1V8 History and Philosophy (with Year Abroad and a term in Venice)
- Year 3 of UHIA-V1V7 Undergraduate History and Philosophy (with a term in Venice)
- UPHA-V700 Undergraduate Philosophy
 - Year 2 of V700 Philosophy
 - Year 2 of V700 Philosophy
 - Year 3 of V700 Philosophy
 - Year 3 of V700 Philosophy
- Year 4 of UPHA-V701 Undergraduate Philosophy (with Intercalated year)
- Year 4 of UPHA-V702 Undergraduate Philosophy (with Work Placement)
- UPHA-V7ML Undergraduate Philosophy, Politics and Economics
 - Year 2 of V7ML Philosophy, Politics and Economics (Tripartite)
 - Year 2 of V7ML Philosophy, Politics and Economics (Tripartite)
 - Year 2 of V7ML Philosophy, Politics and Economics (Tripartite)
 - Year 3 of V7ML Philosophy, Politics and Economics (Tripartite)

- Year 3 of V7ML Philosophy, Politics and Economics (Tripartite)
- Year 3 of V7ML Philosophy, Politics and Economics (Tripartite)
- Year 4 of UPHA-V7MM Undergraduate Philosophy, Politics and Economics (with Intercalated year)
- UPHA-V7MW Undergraduate Politics, Philosophy and Law
 - Year 2 of V7MW Politics, Philosophy and Law
 - Year 2 of V7MW Politics, Philosophy and Law
 - Year 3 of V7MW Politics, Philosophy and Law
 - Year 3 of V7MW Politics, Philosophy and Law
- Year 4 of UPHA-V7MX Undergraduate Politics, Philosophy and Law (with Intercalated Year)

This module is Unusual option for:

- UPHA-L1CA Undergraduate Economics, Psychology and Philosophy
 - Year 2 of L1CA Economics, Psychology and Philosophy
 - Year 2 of L1CC Economics, Psychology and Philosophy (Behavioural Economics Pathway)
 - Year 3 of L1CA Economics, Psychology and Philosophy
- UPHA-V7ML Undergraduate Philosophy, Politics and Economics
 - Year 2 of V7MR Philosophy, Politics and Economics (Bipartite with Economics Major)
 - Year 2 of V7MP Philosophy, Politics and Economics (Bipartite)
 - Year 2 of V7MP Philosophy, Politics and Economics (Bipartite)
 - Year 2 of V7ML Philosophy, Politics and Economics (Tripartite)
 - Year 2 of V7ML Philosophy, Politics and Economics (Tripartite)
 - Year 2 of V7ML Philosophy, Politics and Economics (Tripartite)
 - Year 3 of V7MR Philosophy, Politics and Economics (Bipartite with Economics Major)
 - Year 3 of V7MP Philosophy, Politics and Economics (Bipartite)
 - Year 3 of V7MP Philosophy, Politics and Economics (Bipartite)
- UPHA-V7MM Undergraduate Philosophy, Politics and Economics (with Intercalated year)
 - Year 4 of V7MS Philosophy, Politics and Economics (Bipartite with Economics Major) (with Intercalated Year)
 - Year 4 of V7MS Philosophy, Politics and Economics (Bipartite with Economics Major) (with Intercalated Year)
 - Year 4 of V7MQ Philosophy, Politics and Economics (Bipartite) with Intercalated Year
 - Year 4 of V7MM Philosophy, Politics and Economics (Tripartite) (with Intercalated year)
- UPHA-V7MW Undergraduate Politics, Philosophy and Law
 - Year 2 of V7MW Politics, Philosophy and Law
 - Year 2 of V7MW Politics, Philosophy and Law
 - Year 3 of V7MW Politics, Philosophy and Law
 - Year 3 of V7MW Politics, Philosophy and Law
- Year 4 of UPHA-V7MX Undergraduate Politics, Philosophy and Law (with Intercalated Year)

This module is Core option list A for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
 - Year 3 of GV17 Mathematics and Philosophy
 - Year 3 of GV17 Mathematics and Philosophy

- Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
 - Year 2 of GV17 Mathematics and Philosophy
 - Year 2 of GV17 Mathematics and Philosophy
 - Year 2 of GV17 Mathematics and Philosophy
- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list C for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list F for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
 - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
 - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year

This module is Option list A for:

- UPHA-VL78 BA in Philosophy with Psychology
 - Year 2 of VL78 Philosophy with Psychology
 - Year 3 of VL78 Philosophy with Psychology
- Year 4 of UPHA-VL79 BA in Philosophy with Psychology (with Intercalated year)

This module is Option list B for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
 - Year 2 of G105 Mathematics (MMath) with Intercalated Year
 - Year 4 of G105 Mathematics (MMath) with Intercalated Year
 - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
 - Year 2 of G100 Mathematics
 - Year 2 of G100 Mathematics
 - Year 2 of G100 Mathematics
 - Year 3 of G100 Mathematics
 - Year 3 of G100 Mathematics
 - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
 - Year 2 of G100 Mathematics
 - Year 2 of G103 Mathematics (MMath)
 - Year 2 of G103 Mathematics (MMath)
 - Year 3 of G100 Mathematics

- Year 3 of G103 Mathematics (MMath)
- Year 3 of G103 Mathematics (MMath)
- Year 4 of G103 Mathematics (MMath)
- Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
 - Year 2 of G106 Mathematics (MMath) with Study in Europe
 - Year 4 of G106 Mathematics (MMath) with Study in Europe
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- UMAA-G101 Undergraduate Mathematics with Intercalated Year
 - Year 2 of G101 Mathematics with Intercalated Year
 - Year 4 of G101 Mathematics with Intercalated Year
- UPHA-VQ72 Undergraduate Philosophy and Literature
 - Year 2 of VQ72 Philosophy and Literature
 - Year 3 of VQ72 Philosophy and Literature
- Year 4 of UPHA-VQ73 Undergraduate Philosophy and Literature with Intercalated Year
- Year 2 of UPHA-VQ52 Undergraduate Philosophy, Literature and Classics
- UPHA-V7ML Undergraduate Philosophy, Politics and Economics
 - Year 2 of V7MP Philosophy, Politics and Economics (Bipartite)
 - Year 2 of V7MP Philosophy, Politics and Economics (Bipartite)

This module is Option list C for:

- Year 3 of UHIA-V1V5 Undergraduate History and Philosophy
- Year 4 of UHIA-V1V6 Undergraduate History and Philosophy (with Year Abroad)
- UPHA-V7ML Undergraduate Philosophy, Politics and Economics
 - Year 3 of V7MP Philosophy, Politics and Economics (Bipartite)
 - Year 3 of V7MP Philosophy, Politics and Economics (Bipartite)
 - Year 3 of V7ML Philosophy, Politics and Economics (Tripartite)
 - Year 3 of V7ML Philosophy, Politics and Economics (Tripartite)
 - Year 3 of V7ML Philosophy, Politics and Economics (Tripartite)
- UPHA-V7MM Undergraduate Philosophy, Politics and Economics (with Intercalated year)
 - Year 4 of V7MS Philosophy, Politics and Economics (Bipartite with Economics Major) (with Intercalated Year)
 - Year 4 of V7MS Philosophy, Politics and Economics (Bipartite with Economics Major) (with Intercalated Year)
 - Year 4 of V7MQ Philosophy, Politics and Economics (Bipartite) with Intercalated Year
 - Year 4 of V7MM Philosophy, Politics and Economics (Tripartite) (with Intercalated year)

This module is Option list D for:

- UHIA-V1V5 Undergraduate History and Philosophy
 - Year 2 of V1V5 History and Philosophy
 - Year 3 of V1V5 History and Philosophy

- Year 4 of UHIA-V1V8 Undergraduate History and Philosophy (with Year Abroad and a term in Venice)
- Year 4 of UHIA-V1V6 Undergraduate History and Philosophy (with Year Abroad)
- UHIA-V1V7 Undergraduate History and Philosophy (with a term in Venice)
 - Year 2 of V1V7 History and Philosophy (with a term in Venice)
 - Year 3 of V1V7 History and Philosophy (with a term in Venice)
- UPHA-V7ML Undergraduate Philosophy, Politics and Economics
 - Year 2 of V7MR Philosophy, Politics and Economics (Bipartite with Economics Major)
 - Year 3 of V7MR Philosophy, Politics and Economics (Bipartite with Economics Major)