MA4M6-15 Category Theory

23/24

Department
Warwick Mathematics Institute
Level
Undergraduate Level 4
Module leader
Emanuele Dotto
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Mathematical structures come equipped with a notion of "maps", or "morphisms", which are used to compare objects with such structure. For example, we may not want to distinguish between the sets {1,2,3} and {a,b,c} by noticing that we can map the elements of the first set to the second bijectively.
This principle extends to virtually all mathematical objects. To name a few: one studies groups via group homomorphisms, vector spaces via linear maps, spaces via continuous maps, manifolds via smooth maps, probability spaces via measurable functions, paths via homotopies.

A category consists of a collection of objects, a collection of morphisms, and a composition rule. Many common mathematical constructions only use the structure of a category, for example coproducts, direct sums, kernels, quotients, coinvariants, compactifications. The theme of the module will be to investigate how vaguely analogous constructions in various areas of mathematics are in fact instances of the same construction carried out at the level of a category.

Our primary reference will be "Category theory in context" by Emily Riehl.

The module will cover the notions of categories, functors, natural transformations, limits and colimits, adjunctions, and Freyd's adjoint functor theorem. We will then apply the category theoretical framework to formulate the Seifert Van-Kampen theorem for the fundamental groupoid, and use it to calculate the fundamental group of the circle.

Module aims

To give students a grounding in the theory of category theory, and gain insight into the use of its language throughout areas of mathematics.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

-Categories, functors and natural transformations
-Representable functors and the Yoneda Lemma
-Limits and colimits
-Adjunctions and Freyd's Theorem
-The fundamental groupoid

Learning outcomes

By the end of the module, students should be able to:

- Explain the definitions and properties of the basic notions of category theory
- Understand adjunctions and the importance of Freyd's adjoint functor theorem
- Use the framework of category theory to prove results by universal property
- Recognise common constructions as instances of categorical constructions
- Describe the fundamental groupoid of a gluing of spaces from that of its pieces

Interdisciplinary

Category theory is being increasingly applied in computer science, for example in semantics and functional programming . This module would provide a solid foundation for venturing into this area of modern computer science.

Subject specific skills

Ability to apply categorical principles to various areas of mathematics
Ability to work in the abstract formalism of category theory

Transferable skills

Ability to translate scientific ideas into mathematical language
Ability to communicate complex ideas and mathematical results clearly
Ability to analyse and solve abstract mathematical problems

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (32%)
Seminars	9 sessions of 1 hour (10%)
Private study	55 hours (59%)
Total	94 hours

Private study description

Working on assignments, going over lecture notes, text books, exam revision.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time
Assignments	15%	20 hours
In-person Examination	85%	36 hours
Standard 3 hour written exam.		

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	
Standard 3 hour written exam.		

- Answerbook Gold (24 page)

Feedback on assessment

Written feedback on the outcome of assignments.

Past exam papers for MA4M6

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)

This module is Option list A for:

- Year 2 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Option list B for:

- Year 2 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 4 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 5 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)

This module is Option list C for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
    - Year 4 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- Year 4 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe