MA4L7-15 Algebraic Curves

23/24

Department
  Warwick Mathematics Institute
Level
  Undergraduate Level 4
Module leader
  Rob Silversmith
Credit value
  15
Module duration
  10 weeks
Assessment
  Multiple
Study location
  University of Warwick main campus, Coventry

Description

Introductory description

N/A

[Module web page]

Module aims

The Module will provide students with a foundation in the theory of algebraic curves. The theory is motivated by examples of projective curves of low degree, including conics, plane cubic curves (which relate to Elliptic Curves) and plane quartic curves. The theory of divisors and the Riemann-Roch Theorem provide the technical backbone of the subject (and with its applications a large part of the content of the module) and relate the algebra of rational functions on a curve with their geometric properties and morphisms.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

The module covers basic questions on algebraic curves. The first sections establishes the class of

nonsingular projective algebraic curves in algebraic geometry as an object of study, and, for comparison and motivation, the parallel world of compact Riemann surfaces. After these preliminaries, most of the rest of the course focuses on the Riemann--Roch space L(C,D), the vector space of meromorphic functions on a compact Riemann surface or a nonsingular projective algebraic curve with poles bounded by a divisor D - roughly speaking, allowing more poles gives more meromorphic functions.
The statement of the Riemann-Roch theorem
$\dim L(C,D) \geq 1 - g + \deg D$.

It comes with sufficient conditions for equality. The main thrust of the result is to provide rational functions that allows us to embed C into projective space Pn. The formula involves an invariant called the genus g(C) of the curve. In intuitive topological terms, we think of it as the "number of holes". However, it has many quite different characterisations in analysis and in algebraic geometry, and is calculated in many different ways. The logical relations between these treatments is a little complicated. A middle section of the course emphasizes the meaning and purpose of the theorem (independent of its proof), and give important examples of its applications. The proof of RR is based on commutative algebra. Algebraic varieties have many different types of rings associated with them, including affine coordinate rings, homogeneous coordinate rings, their integral closures, and their localisations such as the DVRs that correspond to points of a nonsingular curve. Footnote to the course notes include (as nonexaminable material) references to high-brow ideas such as coherent sheaves and their cohomology and Serre--Grothendieck duality.

Learning outcomes

By the end of the module, students should be able to:

- Demonstrate understanding of the basic concepts, theorems and calculations related to projective curves defined by homogeneous polynomials of low degree.
- Demonstrate understanding of the basic concepts, theorems and calculations that relate the zeroes and poles of rational functions with the general theory of discrete valuation rings and divisors on projective curves.
- Demonstrate knowledge and understanding of the statement of the Riemann-Roch theorem and an understanding of some of its applications.
- Demonstrate understanding of the proof of the Riemann-Roch theorem.

Indicative reading list

Frances Kirwan, Complex algebraic curves, LMS student notes
William Fulton, Algebraic Curves: An Introduction to Algebraic Geometry online at www.math.lsa.umich.edu/~wfulton/CurveBook.pdf
I.R. Shafarevich, Basic Algebraic Geometry (especially Part 1, Chapter 3, Section 3.7)
Robin Hartshorne, Algebraic Geometry, (Chapter 4 only)

Subject specific skills

Understand both theoretical and practical aspects of algebraic curves, including particular classes of curves and their projective geometry and the meaning and applications of the Riemann-Roch Theorem.

Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

111 hours Independent study, non-assessed example sheets and revision for exam.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D1

	Weighting	Study time	Eligible for self-certification
Assessed worksheets	15%		No
In-person Examination	85%		No

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

Feedback on assessment

Marked coursework and exam feedback.

Past exam papers for MA4L7

Availability

Courses

This module is Optional for:

- TMAA-G1PE Master of Advanced Study in Mathematical Sciences
  - Year 1 of G1PE Master of Advanced Study in Mathematical Sciences
  - Year 1 of G1PE Master of Advanced Study in Mathematical Sciences
- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)

This module is Option list A for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
  - Year 1 of G1PC Mathematics (Diploma plus MSc)
  - Year 2 of G1PC Mathematics (Diploma plus MSc)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Option list B for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
  - Year 1 of G1PC Mathematics (Diploma plus MSc)
  - Year 2 of G1PC Mathematics (Diploma plus MSc)
- Year 4 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 5 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)

- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Option list C for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 3 of G105 Mathematics (MMath) with Intercalated Year
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
  - Year 4 of G106 Mathematics (MMath) with Study in Europe