# MA3K0-15 High-Dimensional Probability

## 23/24

## **Department**

Warwick Mathematics Institute

#### Level

**Undergraduate Level 3** 

#### Module leader

Stefan Adams

#### **Credit value**

15

#### **Assessment**

Multiple

## **Study location**

University of Warwick main campus, Coventry

# **Description**

# Introductory description

N/A

Module web page

## Module aims

- Concentration of measure problem in high dimensions
- · Three basic concentration inequalities
- Application of basic variational principles
- · Concentration of the norm
- · Dependency structures
- · Introduction to random matrices

## **Outline syllabus**

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

• Preliminaries on Random Variables (limit theorems, classical inequalities, Gaussian models,

Monte Carlo)

- Basic Information theory (entropy; Kull-Back Leibler information divergence)
- Concentrations of Sums of Independent Random Variables
- Random Vectors in High Dimensions
- Random Matrices
- Concentration with Dependency structures
- Deviations of Random Matrices and Geometric Consequences
- · Graphical models and deep learning

## Learning outcomes

By the end of the module, students should be able to:

- Understand the concentration of measure problem in high dimensions
- Be able to distinguish three basic concentration inequalities
- Be able to distinguish between concentration for independent families as well as for various dependency structures
- Understanding of basic concentrations of the norm
- Familiar with random matrices (main properties)
- Be familiar with some application of graphical models
- Be able to understand basic variational problems

# Indicative reading list

We won't follow a particular book and will provide lecture notes. The course is based on the following three books where the majority is taken from [1]:

- [1] Roman Vershynin, High-Dimensional Probability: An Introduction with Applications in Data Science, Cambridge Series in Statistical and Probabilistic Mathematics, (2018).
- [2] Kevin P. Murphy, Machine Learning A Probabilistic Perspective, MIT Press (2012).
- [3] Simon Rogers and Mark Girolami, A first course in Machine Learning, CRC Press (2017).
- [4] Alex Kulesza and Ben Taskar, Determinantal point processes for machine learning, Lecture Notes (2013).

## Subject specific skills

- Understanding that the concentration of measure problem requires analytical expertise as well as some basic probability
- Be able to distinguish three basic concentration inequalities
- Be able to distinguish between concentration for independent families as well as for various dependency structures
- Understanding of basic concentrations of the norm
- Familiar with random matrices (main properties)
- Be familiar with some application of graphical models

## Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to

# **Study**

# Study time

Type	Required

Lectures 30 sessions of 1 hour (77%)
Seminars 9 sessions of 1 hour (23%)

Total 39 hours

# Private study description

No private study requirements defined for this module.

# **Costs**

No further costs have been identified for this module.

## **Assessment**

You do not need to pass all assessment components to pass the module.

# **Assessment group D1**

	Weighting	Study time
Written homework	15%	
Written homework (4 example sheets)		
In-person Examination	85%	

Answerbook Gold (24 page)

# Assessment group R

In-person Examination - Resit Weighting Study time
100%

Answerbook Gold (24 page)

## Feedback on assessment

Marked homework and exam feedback.

Past exam papers for MA3K0

# **Availability**

## **Courses**

This module is Optional for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
  - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics

## This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

## This module is Core option list D for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
  - Year 1 of G1PC Mathematics (Diploma plus MSc)
  - Year 2 of G1PC Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 3 of G105 Mathematics (MMath) with Intercalated Year
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
  - Year 4 of G106 Mathematics (MMath) with Study in Europe
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 3 of GG14 Mathematics and Statistics
  - Year 3 of GG14 Mathematics and Statistics
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics, Operational Research, Statistics, Economics (with Intercalated Year)

#### This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

#### This module is Option list E for:

- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- USTA-G301 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics (with Intercalated
  - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)