

# MA3H7-15 Control Theory

**23/24**

**Department**

Warwick Mathematics Institute

**Level**

Undergraduate Level 3

**Module leader**

Tim Sullivan

**Credit value**

15

**Module duration**

10 weeks

**Assessment**

Multiple

**Study location**

University of Warwick main campus, Coventry

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## Description

### Introductory description

The module gives an introduction to control theory. The emphasis is on theory, i.e. mathematics, rather than application. For applications, best to look at Engineering modules. Nevertheless, it is motivated by practical questions. Can one control the state of a system to a given place if the control force is only on some components? Can one stabilise it by suitable feedback control? Can one infer the state of the system from observations of only some components? How to achieve a given effect with minimum control cost? We will formulate these questions mathematically and answer them, at least in simple settings. The module will make use of a wide variety of mathematics, some of which you will already know (linear algebra, ordinary differential equations) and some you might not yet have met (optimisation with constraints, Bayesian inference). I'll give rapid summaries of the things that I expect you to know and more detailed surveys of required topics that are likely to be new to you.

[Module web page](#)

### Module aims

The aim of the module is to show how, as a result of extensive interests of mathematicians, control theory has developed from being a theoretical basis for control engineering into a versatile and active branch of applied mathematics.

## Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Will include the study of controllability, stabilization, observability, filtering and optimal control. Furthermore connections between these concepts will also be studied. Both linear and nonlinear systems will be considered. The module will comprise six chapters. The necessary background material in linear algebra, differential equations and probability will be developed as part of the course.

1. Introduction to Key Concepts.
2. Background Material.
3. Controllability.
4. Stabilization.
5. Observability and Filtering.
6. Optimal Control.

## Learning outcomes

By the end of the module, students should be able to:

- Explain and exploit role of controllability matrix in linear control systems.
- Explain and exploit stabilization for linear control systems.
- Derive and analyze the Kalman filter.
- Understand linear ODEs and stability theory.
- Understand and manipulate Gaussian probability distributions.
- Understand basic variational calculus for constrained minimization in Hilbert space.

## Indicative reading list

E. D. Sontag, Mathematical Control Theory, Texts in Applied Mathematics No 6, Springer Verlag, 1990.

J. Zabczyk, Mathematical Control Theory: An Introduction, Systems and Control, Birkhauser, 1992.

## Subject specific skills

Ability to decide whether a linear system is controllable, observable. Ability to design Kalman filters. Ability to design optimal control.

## Transferable skills

Understanding of how mathematics relates to real-world engineering.

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## Study

## Study time

<b>Type</b>	<b>Required</b>
Lectures	30 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

### **Private study description**

Review lectured material and work on set exercises.

### **Costs**

No further costs have been identified for this module.

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### **Assessment**

You do not need to pass all assessment components to pass the module.

#### **Assessment group B1**

	<b>Weighting</b>	<b>Study time</b>
In-person Examination A 3-hour written exam.	100%	

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- Answerbook Gold (24 page)

#### **Assessment group R**

	<b>Weighting</b>	<b>Study time</b>
In-person Examination - Resit	100%	

- Answerbook Gold (24 page)

### **Feedback on assessment**

Support classes, work returned after marking and exam feedback.

[Past exam papers for MA3H7](#)

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# Availability

## Courses

This module is Optional for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
  - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
- Year 3 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 4 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)

This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
  - Year 1 of G1PC Mathematics (Diploma plus MSc)

- Year 2 of G1PC Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 3 of G105 Mathematics (MMath) with Intercalated Year
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
  - Year 4 of G106 Mathematics (MMath) with Study in Europe
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
  - Year 3 of GF13 Mathematics and Physics
  - Year 3 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 3 of FG31 Mathematics and Physics (MMathPhys)
  - Year 3 of FG31 Mathematics and Physics (MMathPhys)
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 3 of GG14 Mathematics and Statistics
  - Year 3 of GG14 Mathematics and Statistics
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics, Operational Research, Statistics, Economics (with Intercalated Year)

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

This module is Option list E for:

- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- USTA-G301 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics (with Intercalated
  - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)