MA3H2-15 Markov Processes and Percolation Theory

23/24

Department
Warwick Mathematics Institute
Level
Undergraduate Level 3
Module leader
Oleg Zaboronski
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

This module provides an introduction to continuous-time Markov processes and percolation theory, which have numerous applications: random growth models (sand-pile models), Markov decision processes, communication networks.
The module first introduces the theory of Markov processes with continuous time parameter running on graphs. An example of a graph is the two-dimensional integer lattice and an example of a Markov process is a random walk on this lattice. Very interesting problems of such processes involve spatial disorder and dependencies (e.g. burning forests). Therefore, after the main part, an elementary introduction to percolation theory will be given which can be used to study such questions.
Percolation is a simple probabilistic model for spatial disorder, and in physics, chemistry and materials science, percolation concerns the movement and filtering of fluids through porous materials. Recent applications include for example percolation of water through ice which is important for the melting of the ice caps.

Module web page

Module aims

The basic mathematical methods and techniques of random processes and an overview of the most important applications will enable the student to use analytical techniques and models to study questions in modern applications in biological and physical systems, communication networks, financial market, decision processes.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Fundamental concepts of probability theory such as probability space and random variables. Random walks on graphs, and the related theory of electrical networks. Markov Chains, and their convergence to a stationary distribution. Galton-Watson trees. Percolation theory: the phase transition, and determining the threshold in particular cases.

Learning outcomes

By the end of the module, students should be able to:

- The module will enhance the students' knowledge of a foundational core of pure mathematics and an understanding of a range of techniques.
- Research skills will be developed as well in the final two weeks.

Indicative reading list

H.O. Georgii: Stochastics: introduction to probability theory and statistics, de Gruyter (2008). [basic introduction to stochastics and Markov chains (discrete time)]
J. Norris: Markov chains, Cambridge University Press [standard reference treating the topic with mathematical rigor and clarity, and emphasizing numerous applications to a wide range of subjects]
G. Grimmett, D. Stirzaker: Probability and Random Processes, OUP Oxford (2001) [chapter 6 on Markov chains]
G. Grimmett: Probability on Graphs, Cambridge University Press (2010). [Available Online, contains a nice introduction to processes on graphs and percolation]
B. Bollabás, O. Riordan: Percolation, Cambridge University Press (2006). [a modern treatment of percolation. The introduction and the chapter on basic techniques are relevant for the lecture]
G. Grimmett: Percolation, 2nd ed., Springer (1999). [the standard reference on percolation. It contains much more than covered in the lecture. The first two chapters are relevant for the lecture]

Subject specific skills

The module will enhance the students' ability to think clearly, learn new ideas quickly, and construct and follow logical arguments.

Transferable skills

Following complex reasoning, and manipulating precise and intricate concepts.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You must pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group B

	Weighting	Study time
In-person Examination	100%	

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Gold (24 page)

Feedback on assessment

Exam feedback.

Past exam papers for MA3H2

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UCSA-G4G1 Undergraduate Discrete Mathematics
    - Year 3 of G4G1 Discrete Mathematics
    - Year 3 of G4G1 Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
- Year 3 of UMAA-GL11 Undergraduate Mathematics and Economics

This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
    - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
    - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
    - Year 1 of G1PC Mathematics (Diploma plus MSc)
    - Year 2 of G1PC Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)

    - Year 3 of G105 Mathematics (MMath) with Intercalated Year
    - Year 4 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G100 Mathematics
    - Year 3 of G103 Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
    - Year 3 of G106 Mathematics (MMath) with Study in Europe
    - Year 4 of G106 Mathematics (MMath) with Study in Europe
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
    - Year 3 of GG14 Mathematics and Statistics
    - Year 3 of GG14 Mathematics and Statistics
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
    - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
    - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics,Operational Research,Statistics,Economics (with Intercalated Year)

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

This module is Option list E for:

- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 3 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and

Economics (with Intercalated

- Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)