MA3G6-15 Commutative Algebra

23/24

Department
Warwick Mathematics Institute
Level
Undergraduate Level 3
Module leader
Diane Maclagan
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Commutative Algebra is the study of commutative rings, and their modules and ideals. This theory has developed over the last 150 years not just as an area of algebra considered for its own sake, but as a tool in the study of two enormously important branches of mathematics: algebraic geometry and algebraic number theory. The unification which results, where the same underlying algebraic structures arise both in geometry and in number theory, has been one of the crowning glories of twentieth century mathematics and still plays an absolutely fundamental role in current work in both these fields.
One simple example of this unification will be familiar already to anyone who has noticed the strong parallels between the ring Z (a Euclidean Domain and hence also a Unique Factorization Domain) and the ring F[X] of polynomials over a field (which has both the same properties). More generally, the rings of algebraic integers which have been studied since the 19th century to solve problems in number theory have parallels in rings of functions on curves in geometry.
While self-contained, this course will also serve as a useful introduction to either algebraic geometry or algebraic number theory.

Module web page

Module aims

Commutative Algebra is the study of commutative rings, and their modules and ideals. The

module Commutative Algebra is to build bridges connecting algebra, geometry, and number theory. Also it serves as a preliminary module for both algebraic geometry and algebraic number theory.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Topics: Gröbner bases, modules, localization, integral closure, primary decomposition, valuations and dimension.

Learning outcomes

By the end of the module, students should be able to:

- understand the Hilbert bases theorem, know several examples of Noetherian rings.
- be able to compute the Gröbner bases of an ideal of the polynomial ring; Spec of some rings; zero locus of ideals.
- understand the proof of Cayley-Hamilton theorem, and to be able to apply it to prove Nakayama Lemma, Tower's law and other theorems in module theory.
- be able to determine algebraic integers via several methods in the theory of finite extensions.
- be able to compute the localisation, integral closure, dimension of rings.
- understand the proof of Hilbert Nullstellensatz, and to be able to apply it to compute the primary decomposition of ideals in a polynomial ring.

Indicative reading list

M.F. Atiyah, I.G. MacDonald, Introduction to Commutative Algebra. Addison-Wesley 1969; reprinted by Perseus 2000. [QA251.3.A8]
D. Eisenbud, Commutative algebra with a view toward algebraic geometry. Springer 1995. [QA251.3.E4]
M. Reid, Undergraduate Commutative Algebra. CUP 1995. [QA251.3.R3]
R.Y. Sharp, Steps in Commutative Algebra (2nd ed.) CUP 2000. [QA251.3.S4]
O. Zariski and P. Samuel, Commutative Algebra, (vols I and II). Springer 1975-6. [QA251.3.Z2]

Subject specific skills

The students will get their first knowledge in algebraic geometry and algebraic number theory. The students will train their programming skill in various assignments, especially computational problems for Gröbner bases.

Transferable skills

The students will acquire confidence in dealing with apparently complicated problems which nonetheless have a simple underlying solution.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group D1

	Weighting	Study time
Coursework	15%	
In-person Examination	85%	

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Gold (24 page)

Feedback on assessment

Marked coursework and exam feedback.

Past exam papers for MA3G6

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UCSA-G4G1 Undergraduate Discrete Mathematics
    - Year 3 of G4G1 Discrete Mathematics
    - Year 3 of G4G1 Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
- Year 3 of UMAA-GL11 Undergraduate Mathematics and Economics

This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
    - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
    - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
    - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
    - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)

- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
    - Year 1 of G1PC Mathematics (Diploma plus MSc)
    - Year 2 of G1PC Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
    - Year 3 of G105 Mathematics (MMath) with Intercalated Year
    - Year 4 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G100 Mathematics
    - Year 3 of G103 Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
    - Year 3 of G106 Mathematics (MMath) with Study in Europe
    - Year 4 of G106 Mathematics (MMath) with Study in Europe
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
    - Year 3 of GF13 Mathematics and Physics
    - Year 3 of GF13 Mathematics and Physics
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
    - Year 3 of GG14 Mathematics and Statistics
    - Year 3 of GG14 Mathematics and Statistics
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
    - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
    - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics,Operational Research,Statistics,Economics (with Intercalated Year)

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

This module is Option list E for:

- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 3 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
    - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)