

# MA3D5-15 Galois Theory

**23/24**

**Department**

Warwick Mathematics Institute

**Level**

Undergraduate Level 3

**Module leader**

Gavin Brown

**Credit value**

15

**Module duration**

10 weeks

**Assessment**

Multiple

**Study location**

University of Warwick main campus, Coventry

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## Description

### Introductory description

Galois theory is the study of solutions of polynomial equations. You know how to solve the quadratic equation  $ax^2+bx+c=0$  by completing the square, or by that formula involving plus or minus the square root of the discriminant  $b^2-4ac$ . The cubic and quartic equations were solved “by radicals” in Renaissance Italy. In contrast, Ruffini, Abel and Galois discovered around 1800 that there is no such solution of the general quintic. Although the problem originates in explicit manipulations of polynomials, the modern treatment is in terms of field extensions and groups of “symmetries” of fields. For example, a general quintic polynomial over  $\mathbb{Q}$  has five roots  $\alpha_1, \dots, \alpha_5$ , and the corresponding symmetry group is the permutation group  $S_5$  on these.

[Module web page](#)

### Module aims

The course will discuss the problem of solutions of polynomial equations both in explicit terms and in terms of abstract algebraic structures. The course demonstrates the tools of abstract algebra (linear algebra, group theory, rings and ideals) as applied to a meaningful problem.

### Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Developing the algebraic tools to understand the solutions of polynomial equations and further applications. In particular, we cover fields, field extensions, separable and normal extensions, automorphism groups of fields and their extensions, Galois extensions, the Fundamental Theorem of Galois theory, soluble groups and radical extensions, insoluble quintics, impossibility of trisecting an angle and squaring the circle.

## **Learning outcomes**

By the end of the module, students should be able to:

- Solution by radicals of cubic equations and (briefly) of quartic equations.
- The characteristic of a field and its prime subfield. Field extensions as vector spaces.
- Factorisation and ideal theory in the polynomial ring  $k[x]$ ; the structure of a simple field extension.
- The impossibility of trisecting an angle with straight-edge and compass.
- The existence and uniqueness of splitting fields.
- Groups of field automorphisms; the Galois group and the Galois correspondence.
- Radical field extensions; soluble groups and solubility by radicals of equations.
- The structure and construction of finite fields.

## **Indicative reading list**

Books: DJH Garling, A course in Galois theory, CUP.  
IN Stewart, Galois Theory, Chapman and Hall.

## **Subject specific skills**

Fields and their automorphism groups are key objects for anyone continuing to study algebra, number theory, algebraic geometry etc. Students will also learn how to use a group of automorphisms to study an object, this is used throughout pure mathematics, applied mathematics, theoretical physics, and even biology and chemistry (in studying symmetries of molecules).

## **Transferable skills**

Problem solving, logical reasoning and quantitative analytical skills are all tested and improved in this module. Moreover, the students are required to use knowledge from a various range of background modules and link them together to gain understanding of the topics in this module; a key skill used throughout every day and working life.

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## **Study**

## Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

## Private study description

Review lectured material and work on set exercises.

## Costs

No further costs have been identified for this module.

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## Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

### Assessment group D1

	Weighting	Study time
Assessed worksheets and tests	15%	
Best 3 out of 4 assessed worksheet		
In-person Examination	85%	

- Answerbook Gold (24 page)

### Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Gold (24 page)

## Feedback on assessment

Marked coursework and exam feedback.

[Past exam papers for MA3D5](#)

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## Availability

### Courses

This module is Optional for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
  - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
- Year 4 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)

This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)

- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
  - Year 1 of G1PC Mathematics (Diploma plus MSc)
  - Year 2 of G1PC Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 3 of G105 Mathematics (MMath) with Intercalated Year
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
  - Year 4 of G106 Mathematics (MMath) with Study in Europe
- Year 3 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 3 of FG31 Mathematics and Physics (MMathPhys)
  - Year 3 of FG31 Mathematics and Physics (MMathPhys)
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 3 of GG14 Mathematics and Statistics
  - Year 3 of GG14 Mathematics and Statistics
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics, Operational Research, Statistics, Economics (with Intercalated Year)

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

This module is Option list E for:

- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- USTA-G301 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics (with Intercalated)
  - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)