# **MA3D4-15 Fractal Geometry**

#### 23/24

**Department** 

Warwick Mathematics Institute

Level

**Undergraduate Level 3** 

Module leader

Han Yu

Credit value

15

Module duration

10 weeks

**Assessment** 

Multiple

**Study location** 

University of Warwick main campus, Coventry

# **Description**

# Introductory description

Fractals are geometric forms that possess structure on all scales of magnification. Examples are the middle third Cantor set, the von Koch snowflake curve and the graph of a nowhere differentiable continuous function.

The main focus of the module will be the mathematical theory behind fractals, such as the definition and properties of the Hausdorff dimension, which is a number quantifying how ``rough" the fractal is and which reduces to the usual dimension when applied to Euclidean space. However, more recent developments will be included, such as iterated function systems (used for image compression) where we study how a fractal is approximated by other compact subsets.

#### Module web page

#### Module aims

The aim of this module is to give students a better perspective and understanding of the general area of Fractal geometry and its relationship to other aspects of analysis, geometry and dynamical systems, although no background knowledge of these other areas is assumed. More particularly, it is expected that the students will develop skills and techniques that will allow them to study many areas where Fractal Geometry plays a role, including analytic number theory, information theory, engineering and Dynamical Systems and Ergodic Theory.

## **Outline syllabus**

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

The course will cover: Iterated function schemes, Julia sets, Sierpinski triangles, Circle packings, Box Dimension, Hausdorff dimension, Iterated Function Schemes, Moran's Theorem, Potential theoretic methods, Falconer's theorem, Projection and Slice theorems, Multifractal analysis, Number theory applications, including Besicovich's Theorem, Numerical estimates.

## Learning outcomes

By the end of the module, students should be able to:

- By the end of this module students will be familiar with the different constructions of fractal sets, including several explicit constructions, and the different notions of dimension available to describe them. By the end of this module students will be familiar with the different constructions of fractal sets, including several explicit constructions, and the different notions of dimension available to describe them.
- They will also have the knowledge to estimate these values and to quantify the notion of being fractal.
- Students will learn the connection between these sets and values, and other areas of mathematics.

# Indicative reading list

K. Falconer, Fractal geometry: mathematical foundations and applications, Wiley, 1990 or 2003. (We shall cover much of the first half of this book.)

# Subject specific skills

The students will develop skills in real analysis and probability which with be motivated by problems in Fractal geometry, but be invaluable in other areas of pure and applied mathematics.

#### Transferable skills

The students will acquire confidence in dealing with apparently complicated problems which nonetheless have a simple underlying solution.

# **Study**

# Study time

Type Required

Lectures 30 sessions of 1 hour (20%)
Tutorials 9 sessions of 1 hour (6%)

Private study 111 hours (74%)

Total 150 hours

# Private study description

Review lectured material and work on set exercises.

#### Costs

No further costs have been identified for this module.

#### **Assessment**

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

## **Assessment group B1**

	Weighting	Study time
In-person Examination	100%	

Answerbook Gold (24 page)

## Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

Answerbook Gold (24 page)

### Feedback on assessment

Exam feedback

Past exam papers for MA3D4

# **Availability**

#### **Courses**

#### This module is Optional for:

- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
  - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics

#### This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

#### This module is Core option list D for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

#### This module is Option list A for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 3 of G105 Mathematics (MMath) with Intercalated Year
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)

- Year 4 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 3 of GG14 Mathematics and Statistics
  - Year 3 of GG14 Mathematics and Statistics
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics, Operational Research, Statistics, Economics (with Intercalated Year)

#### This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

#### This module is Option list E for:

- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- USTA-G301 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics (with Intercalated
  - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)