MA265-10 Methods of Mathematical Modelling 3

23/24

Department
Warwick Mathematics Institute
Level
Undergraduate Level 2
Module leader
Marie-Therese Wolfram
Credit value
10
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Some modern technology became possible due to computing power and mathematical methods. Examples include search algorithms, data compression, artificial intelligence, etc. This module will develop explicit mathematical methods used in some of these applications.

Module aims

The module gives an introduction to the theory and practice of optimisation as well as the fundamentals of approximation theory.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Recap: necessary and sufficient conditions for local min/max, convex functions and sets, Jensen's inequality, level sets.
2. Differentiable unconstrained problems: existence of local minimisers, Hessian, classification

of critical points.
3. Convex linear and quadratic problems: linear programming, least squares and regression, steepest descent, singular value decomposition, stochastic gradient descent, Gibbs inequalities, entropy minimisation, applications (line search and optimisation in 1D, linear and logistic regression, backward propagation for neural network training, support vector machines).
4. Constrained optimisation: Lagrange multipliers, Karush–Kuhn–Tucker conditions.
5. The discrete Fourier transform: Parseval's theorem, FFT in applications (convolutions in PDEs, signal processing, audio and video compression).
6. Approximation properties of complete orthonormal systems: Hilbert spaces, Fourier basis and classical orthogonal polynomials.
7. Linear least squares: interpolation by algebraic polynomials (Chebyshev polynomials, Lagrange polynomials), interpolation by trigonometric polynomials (Fourier series).

Learning outcomes

By the end of the module, students should be able to:

- understand critical points of multivariable functions
- apply various techniques to solve nonlinear optimisation problems and understand their applications, including to machine learning
- use Lagrange multipliers and the Karush–Kuhn–Tucker conditions to solve constrained nonlinear optimisation problems
- have a working knowledge of discrete Fourier transformation
- understand the basic concepts of Hilbert space theory and its relation to approximation by orthogonal functions, including approximation of functions by algebraic and trigonometric polynomials

Indicative reading list

S. Boyd. 'Convex optimization', Cambridge University Press 2004
Y. Nesterov, 'Lectures on Convex Optimization', Springer, 2018
J. D. Powell, 'Approximation Theory and Methods', Cambridge University Press, 1981
N. Trefethen, 'Approximation Theory and Practice'

Subject specific skills

The module gives an introduction to the theory and practice of optimisation as well as the fundamentals of approximation theory. The students will learn applications of these techniques to a range of modern technologies.

Transferable skills

The algorithmic techniques taught have widespread "real world" applications. Examples include ranking in search engines, linear programming and optimisation, signal analysis and machine learning.

Study

Study time

Type	Required
Lectures	20 sessions of 1 hour (20%)
Online learning (independent)	9 sessions of 1 hour (9%)
Private study	13 hours (13%)
Assessment	58 hours (58%)
Total	100 hours

Private study description

Working on assignments, going over lecture notes, text books, exam revision.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time
Assignments	15%	20 hours
Examination	85%	38 hours

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

Feedback on assessment

Marked homework (both assessed and formative) is returned and discussed in smaller classes. Exam feedback is given.

[Past exam papers for MA265]

Availability

Courses

This module is Core for:

- Year 2 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 2 of G100 Mathematics
  - Year 2 of G100 Mathematics
  - Year 2 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 2 of G100 Mathematics
  - Year 2 of G103 Mathematics (MMath)
  - Year 2 of G103 Mathematics (MMath)
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- Year 2 of UMAA-G101 Undergraduate Mathematics with Intercalated Year