MA258-12 Mathematical Analysis III

23/24

Department
Warwick Mathematics Institute
Level
Undergraduate Level 2
Module leader
Roger Tribe
Credit value
12
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

This is the third module in the series Analysis I, II, III that covers rigorous analysis.
It's core is a theory of integration of functions of one variable.

Integration involves taking a limit, and the deeper properties of integration require a precise and careful analysis of this limiting process. This module proves that every continuous function can be integrated, and proves the fundamental theorem of calculus. It also discusses how integration can be applied to define some of the basic functions of analysis and to establish their fundamental properties.

Many functions can be written as limits of sequences of simpler functions (or as sums of series): thus a power series is a limit of polynomials, and a Fourier series is the sum of a trigonometric series. The second part of the module develops methods for deciding when a function defined as the limit of a sequence of other functions is continuous, differentiable, integrable, and for differentiating and integrating this limit.

The third part of the module gives a brief introduction to norms and inner products, illustrating their use in
problems on Euclidean space and in function spaces, in particular to solvability of differential equation

[Module web page]

Module aims

1. To develop a good working knowledge of the construction of the integral of regulated functions;
2. to study the continuity, differentiability and integral of the limit of a uniformly convergent sequence of functions;
3. to use the concept of norm in a vector space to discuss convergence and continuity there.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

This covers three topics: (1) integration, (2) convergence of sequences and series of functions, (3) Norms.

The idea behind integration is to compute the area under a curve. The fundamental theorem of calculus gives the precise relation between integration and differentiation. However, integration involves taking a limit, and the deeper properties of integration require a precise and careful analysis of this limiting process. This module proves that every continuous function can be integrated, and proves the fundamental theorem of calculus. It also discusses how integration can be applied to define some of the basic functions of analysis and to establish their fundamental properties.

Many functions can be written as limits of sequences of simpler functions (or as sums of series): thus a power series is a limit of polynomials, and a Fourier series is the sum of a trigonometric series with coefficients given by certain integrals. The second part of the module develops methods for deciding when a function defined as the limit of a sequence of other functions is continuous, differentiable, integrable, and for differentiating and integrating this limit. Norms are used at several stages and finally applied to show that a Differential Equation has a solution.

Learning outcomes

By the end of the module, students should be able to:

- Understand the need for a rigorous theory of integration, and that this can be developed for regulated functions by approximating the area under the graph by rectangles;
- understand uniform and pointwise convergence of functions together with properties of the limit function;
- be able to prove the main results of integration: any continuous function can be integrated on a bounded interval and the Fundamental Theorem of Calculus;
- prove and apply the Contraction Mapping Theorem.

Subject specific skills

1. Ability to work rigorously with one dimensional integrals.
2. Ability to work rigorously with functions defined via limits.
3. An understanding of a range of examples of norms and inner products and their uses.

Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (77%)
Tutorials	9 sessions of 1 hour (23%)
Total	39 hours

Private study description

80 hours private study, revision for exams, and assignments

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D2

	Weighting	Study time	Eligible for self-certification
Assignment	15%		No
In-person Examination	85%		No

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

- Answerbook Pink (12 page)

Feedback on assessment

Support classes, marked assignments and exam feedback.

Past exam papers for MA258

Availability

Courses

This module is Core for:

- Year 2 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)

This module is Optional for:

- Year 2 of USTA-G305 Undergraduate Data Science (MSci) (with Intercalated Year)
- Year 3 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics

This module is Option list A for:

- Year 2 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics

This module is Option list E for:

- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
  - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)