# MA257-10 Introduction to Number Theory

## 23/24

**Department**
Warwick Mathematics Institute
**Level**
Undergraduate Level 2
**Module leader**
Sam Chow
**Credit value**
10
**Module duration**
10 weeks
**Assessment**
Multiple
**Study location**
University of Warwick main campus, Coventry

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## Description

### Introductory description

Number theory is an ancient and beautiful subject that investigates the properties of the integers, and other questions motivated by them. For example, one studies questions like:

- do equations have integer solutions? If so, do they have infinitely many integer solutions? How are those solutions distributed, and can we parametrise them? Pythagorean triples, and Fermat's Last Theorem, fall into this general area of Diophantine equations.
- how do special subsets of the integers behave? The primes are probably the best example of a special subset, and one wants to know: are there infinitely many? How are they distributed? Are there interesting patterns amongst them? The most famous unsolved problem in mathematics, the Riemann Hypothesis, is connected with this.

In this module we will explore various topics that underlie the deeper study of the integers, and see some initial applications. In particular, we study:

- factorisation in the integers and in other rings
- congruences and arithmetic mod n, including primitive roots
- quadratic reciprocity

- Diophantine equations, including writing integers as sums of squares
- more advanced topics (e.g. encryption, primality checking, basic analytic prime number theory)

Module web page

### Module aims

To introduce students to elementary number theory and provide a firm foundation for later number theory and algebra modules.

### Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- Factorisation, divisibility, Euclidean Algorithm, Chinese Remainder Theorem.
- Congruences. Structure on Z/mZ and its multiplicative group. Theorems of Fermat and Euler. Primitive roots.
- Quadratic reciprocity, Diophantine equations.
- Elementary factorization algorithms.
- Introduction to Cryptography.
- Geometry of numbers, sum of two and four squares.

### Learning outcomes

By the end of the module, students should be able to:

- Work with prime factorisations of integers
- Solve congruence conditions on integers
- Determine whether an integer is a quadratic residue modulo another integer
- Apply geometry of numbers methods to solve some Diophantine equations
- Follow advanced courses on number theory in the third year

### Indicative reading list

H. Davenport, The Higher Arithmetic, Cambridge University Press.

G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers, Oxford University Press, 1979.

K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, Springer-Verlag, 1990.

### Subject specific skills

By the end of the module the student should be able to:

- work with prime factorisations of integers
- solve congruence conditions on integers
- determine whether an integer is a quadratic residue modulo another integer
- apply geometry of numbers methods to solve some Diophantine equations
- follow advanced courses on number theory in the third and fourth year

### Transferable skills

The module will help to develop skills in understanding, assessing and constructing logical arguments (especially of a quantitative nature), and presenting these clearly in writing.

Some parts of the module will explore the difference between a theoretical solution of a problem and a solution that can be practically implemented with current computing resources, a distinction that is crucial in many real world applications of mathematical concepts.

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## Study

## Study time

| Type | Required |
|---|---|
| Lectures | 20 sessions of 1 hour (20%) |
| Tutorials | 9 sessions of 1 hour (9%) |
| Private study | 71 hours (71%) |
| Total | 100 hours |

### Private study description

Review lectured material and work on set exercises.

## Costs

No further costs have been identified for this module.

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## Assessment

You do not need to pass all assessment components to pass the module.

### Assessment group D

| | Weighting | Study time |
|---|---|---|
| Assignment | 15% | |

| | Weighting | Study time |
|---|---|---|
| Examination | 85% | |

- Answerbook Pink (12 page)

### Assessment group R

| | Weighting | Study time |
|---|---|---|
| In-person Examination - Resit | 100% | |

- Answerbook Pink (12 page)

### Feedback on assessment

Support Classes
Marked homework will be returned to students.
Exam feedback.

Past exam papers for MA257

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## Availability

## Courses

This module is Core option list A for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list B for:

- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 2 of G4G1 Discrete Mathematics
  - Year 2 of G4G1 Discrete Mathematics
- Year 2 of UCSA-G4G3 Undergraduate Discrete Mathematics
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 2 of G105 Mathematics (MMath) with Intercalated Year
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 2 of G100 Mathematics
  - Year 2 of G100 Mathematics
  - Year 2 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 2 of G100 Mathematics
  - Year 2 of G103 Mathematics (MMath)
  - Year 2 of G103 Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 2 of GG14 Mathematics and Statistics
  - Year 2 of GG14 Mathematics and Statistics
- UMAA-G101 Undergraduate Mathematics with Intercalated Year
  - Year 2 of G101 Mathematics with Intercalated Year
  - Year 4 of G101 Mathematics with Intercalated Year

This module is Option list B for:

- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
  - Year 2 of GF13 Mathematics and Physics
  - Year 2 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 2 of FG31 Mathematics and Physics (MMathPhys)
  - Year 2 of FG31 Mathematics and Physics (MMathPhys)
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
  - Year 2 of Y602 Mathematics,Operational Research,Stats,Economics
  - Year 2 of Y602 Mathematics,Operational Research,Stats,Economics