MA254-12 Theory of ODEs

23/24

Department
Warwick Mathematics Institute
Level
Undergraduate Level 2
Module leader
James Robinson
Credit value
12
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Many fundamental problems in the applied sciences reduce to understanding solutions of ordinary differential equations (ODEs). Examples include the laws of Newtonian mechanics, predator-prey models in Biology, and non-linear oscillations in electrical circuits, to name only a few. These equations are often too complicated to solve exactly, so one tries to understand qualitative features of solutions.
Some questions we will address in this course include:
When do solutions of ODEs exist and when are they unique? What is the long time behaviour of solutions and can they "blow-up" in finite time? These questions culminate in the famous Picard-Lindelof theorem on existence and uniqueness of solutions of ODEs.
The main part of the course will focus on phase space methods. This is a beautiful geometrical approach which often enables one to understand the behaviour of solutions near critical points - often exactly the regions one is interested in. Different trajectories will be classified and we will develop techniques to answer important questions on the stability properties (or lack thereof) of given solutions.
We will eventually apply these powerful methods to particular examples of practical importance, including the Lotka-Volterra model for the competition between two species and to the Van der Pol and Lienard systems of electrical circuits.
The course will end with a discussion of the Sturm-Liouville theory for solving boundary value problems.

Module web page

Module aims

Extend the knowledge of first year ODEs with a mixture of applications, modelling and theory to prepare for more advanced modules later on in the course.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Introduction: The module will begin with the introduction of a few model systems to motivate questions and techniques; which will reappear throughout the module, applying the new techniques as they are acquired. Examples: Lotka-Volterra, Duffing, Lorenz, Hodgkin-Huxley and Fitzhugh-Nagumo, general Hamiltonian systems / nonlinear oscillator, general gradient flows.

Part I: Theory of Initial Value Problems

1. Picard Thm in R^n: concept of well-posedness, local existence and uniqueness, non-uniqueness, maximal existence interval, blowup
2. Linear theory in R^n: general solutions for constant coefficients, exponential of a matrix, variation of constants in R^n, Gronwall Lemma
3. Euler's Method: convergence, long-time behaviour

Part II: Qualitative Theory of Initial Value Problems
4. Stability: linear stability, Lyapunov stability, convergence to equilibrium
5. Qualitative Theory in R^2: phase plane analysis, equilibria, local phase portraits (sketch of Hartmann-Grobman Thm), limit cycles, attractors, Bendixson-Dulac, Poincare-Bendixson,
6. Informal introduction to chaos, bifurcation, catastrophe to motivate further modules in dynamical systems (definitions and relation to applications detailed above).

Part III: Theory of Linear Boundary Value Problems (time permitting)
7. Linear 2-point bvps: Fredholm alternative and Green's function, link to Fourier series 8. Sturm-Liouville theory

Learning outcomes

By the end of the module, students should be able to:

- Determine the fundamental properties of solutions to certain classes of ODEs, such as existence and uniqueness of solutions.
- Sketch the phase portrait of 2-dimensional systems of ODEs and classify critical points and trajectories.
- Classify various types of orbits and possible behaviour of general non-linear ODEs.
- Understand the behaviour of solutions near a critical point and how to apply linearization techniques to a non-linear problem.
- Apply these methods to certain physical or biological systems.

Indicative reading list

Elementary Differential Equations and Boundary Value Problems, Boyce DiPrima 1997

Differential Equations, Dynamical Systems, and an Introduction to Chaos, Hirsch, Smale 2003

Nonlinear Systems, Drazin 1992

Subject specific skills

See learning outcomes

Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (75%)
Seminars	10 sessions of 1 hour (25%)
Total	40 hours

Private study description

Private study, preparation, revision for exams, reviewing lectured material and working on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group B1

	Weighting	Study time
In-person Examination	100%	

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Pink (12 page)

Feedback on assessment

Marked assignments and exam feedback.

Past exam papers for MA254

Availability

Courses

This module is Optional for:

- Year 3 of UMAA-GL11 Undergraduate Mathematics and Economics

This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy

This module is Core option list C for:

- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year

This module is Option list A for:

- Year 3 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- Year 2 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 2 of G106 Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
- Year 2 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)

This module is Option list B for:

- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 3 of GG14 Mathematics and Statistics
  - Year 3 of GG14 Mathematics and Statistics
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)